

It is for these reasons that such methods are “quasi-optimal”. Namely, only some of the risk factors are addressed directly, and in any case a 1-dimensional (or low-dimensional) “rank optimisation” will be able provide only a very narrow/limited type of optimisation.

In practice there will be other aspects to the rebalance that will be left to the trader’s expertise. The “best” hedging instrument selection will be affected by several factors, and not all of which are easily accommodated by a “simple optimiser”. One such real world factor will be transactions costs (e.g. bid/offer spread) and closely related to this the liquidity of the hedging instrument(s). Similarly, implications due to “vol skew”, “roll idiosyncrasies”, and other “reality impact” matters will affect the decisions. These issues will be discussed and incorporated into the rebalance methodology in later Chapters and in [8.a]. For now, it is important to note that here the trader is in effect acting as part of the “optimisation process”, as opposed to trying to build these types of “trading knowledge” into some giant/complex machine-based optimiser (as in the previous Sections).

Of course, if it was possible to create a giant rebalance optimiser that was highly reliable and P&L efficient, then there would be no need for traders, so long as your business mandate could afford such machinery, see also Chapter 22.

## 10.7.9 Single-Period Dynamic Hedge Optimisation

This Section uses (mostly) Pyramid Hedging (PH) to illustrate single period rebalances. Again, the idea will be to provide multi-dimension risk/hedging management, but now in some “(quasi) optimal” sense.

Caveat: Some of the discussions here, and indeed throughout the Book, provide “indicative” suggestions for “fine-tuning” rebalances. Those “suggestions” are examples to illustrate reality impact. They MUST NOT be taken as some “carved in stone cookbook recipe” for rebalancing. Some fine-tuning will be appropriate only under certain market conditions, and only for certain types of operating mandates. Indeed, there will be some techniques that are trader/trading style specific. Two traders may achieve the same risk/return performance by using very different methods that each of them have “comfort” with.

Be sure to verify rebalance strategies for your own mandate, perhaps with PaR and test trade audits.

PH relies on “rank optimisation”. This methodology selects the “best” solution by listing all the admissible solutions, and then “ordering” or “ranking” them according to some user desired priority. The resulting “sorted” or “ranked” list will have the “optimal” choices at the “head” of the list. For example, one “best” choice for hedging Gamma might be to list

all the available hedging/rebalancing options, and then listing in order of “best Gamma per dollar premium”.

The idea is to use a quasi-optimiser on a dynamic or high-frequency basis, rather like the “assumed perfect/continuous Delta rebalancing” buried inside the BSM framework. The quasi-optimiser helps alleviate some of the crucial shortcomings of the “theoretical/idealised” assumptions in BSM. For example, it is not possible (or indeed desirable) to apply continuous rebalancing in the real world. Thus, relying purely on a high frequency Delta strategy may experience considerable “slippage”, since the Delta rebalances are always “running behind the market”. A quasi-optimiser is helpful in establishing rebalances that reduce that type of (curvature induced) slippage. In addition, these rebalance methodologies can also “recognise” that BSM ignores certain “reality impact” risks, such as Vega.

Crucially, by moving to a “more than just Delta” high frequency strategy there arises the question of “holistic consistency”. If the BSM options prices do not account for these real world effects, and you are actively rebalancing to manage those effects, then what will that mean for your holding period P&L. For example, have you charged a sufficiently high premium or bid/offer spread to cover the cost of these more involved rebalancing methods? Moreover, does this process still result with a risk-neutral/arbitrage-free valuation mechanism? If not, then how should the valuation/operating parameters be adjusted (assuming they still land inside your mandate). By now it should be no surprise to hear that some combination of PaR and trade audits will be essential in answering such questions, see Chapter 22.

#### 10.7.9.1 Pyramid: Delta/Gamma/Rho

To illustrate a fairly standard Pyramid Hedge targeting a Delta/Gamma/Rho neutral rebalance, consider the 20 contract 110 call contract position from the previous Section. Table 10.7 – 1 shows the target position and the usual Greeks also for the available hedge instruments (of course in the real world the admissible space of hedge instruments would be very much larger). As above, this Table is created with the accompanying software.

# Option Portfolio Scenario Analysis - 1

		Today	01-Jul-10									a/e/f
		Pos Size	Exp Date	D2E	Strike	Type	Funding	Income	Mrkt Price	Vol	XrType	
1	Underlying1		1y	365					100			
2	Underlying2		2y	732					100			
3	Underlying3		6m	186					100			
	Depo Futs	0	1j	76	Only use this (or something like it) if there are no Underlyings above for Rho							
1	Underlying1	20	01-Jul-11	365	110	Call	4.00%	6.00%	1.670	18.00%	e	
1	Underlying1		01-Jul-11	365	120	Call	4.00%	6.00%	8.900	18.00%	e	
3	Underlying3		03-Jan-11	186	115	Call	4.00%	6.00%	4.770	18.00%	e	
2	Underlying2		02-Jul-12	732	99	Put	6.00%	0.00%	17.000	18.00%	e	

Instrument Results								DPY=	365
1	2	3	4	5	6	7	10		
Premium	Delta	Gamma	Rho	Theta	Vega	Imp Vol	Intrinsic		
			25						
2.828	0.2740	0.01794	24.573	-2.2447	32.2864	14.22%	0.000		
1.224	0.1418	0.01223	12.956	-1.6486	22.0133	42.26%	0.000		
0.805	0.1310	0.01639	6.265	-2.3616	15.0374	36.55%	0.000		
4.620	-0.2614	0.01276	-61.696	-0.2214	46.0652	43.31%	0.000		

Table 10.7 – 1. Portfolio showing 20 contract 110 call target position and the underlying and options contracts available for hedging.

The hedge instrument Gammas can be divided by their respective premia<sup>314</sup>. The resulting Gamma/Price or “G/P” values are shown in Table 10.7 – 2.

	21.9
	G/P
1	0.00000
2	0.00137
3	0.00344
4	0.00075
5	0.00000
	0
	3
	0.003437
	0.016394

Table 10.7 – 2. Pyramid Hedge calculation determining the “best” Gamma hedging instrument in terms of Gamma/Premium, showing that the 3<sup>rd</sup> instrument is ranked the highest (0.00344), and based on that the Gamma neutral hedge is 21.9 contracts for the current position.

The first row of the G/P’s is zero (beside the green “1”), since that is the target instrument, so its contribution is zeroed. The value 3<sup>rd</sup> from the bottom indicates the instrument

<sup>314</sup> Here it’s the market input options prices used rather than the BSM calculated premia, since the market values are the expected cost, and that is the basis for the optimisation.