13.1 How to Decided on Early Exercise – Part 1

This Section considers the standard academic no-arbitrage argument for one particular early exercise "trigger strategy". As will be seen, in spite of its legitimate logic, this strategy is imperfect in a real world setting. Nevertheless, it is the market convention assumption. Though, it is the market convention assumption not because of its "trading validity", but rather due to practical considerations on real trading floors. This topic is revisited further below, and examined in detail in [8.c].

For the present, the following particular no-arbitrage consideration will be the basis for the "trigger strategy" for the valuation of early exercise facilities. The basic argument is to consider whether at any point in time the value of the option is less than its intrinsic value. An equivalent statement of this comparison/trigger is to assume that traders would be profit maximising (a standard assumptions inside virtually all financial and economic consideration[358], though one that does not hold-up in reality all the time). Then, under a no-arbitrage assumption, it is argued that traders would always trade in a manner with the best value to them. Thus, if the intrinsic value is greater than the option premium, then sell the option, and buy (i.e. take delivery) of the intrinsic (i.e. the profit from the delivery of the underlying for a price equal to the strike subtracted from the currently traded market price of the underlying). In an academic environment, it is then supposed that (as there are no transactions cost, there is infinite liquidity, etc. in academia), the instantaneous effect of the possibility of "arbing" the option for the intrinsic is that the option's value must become at least that of the intrinsic. This is often expressed as:

$$\begin{aligned} "Max\ (Rational)\ Value" &= Max\left[\underbrace{Intrinsic\ Value}_{Current},\ \underbrace{Option\ Value}_{Expected}\right] \\ &= No - Arbitrage\ (Early\ \ Excercise)\ Option\ Value \end{aligned} \tag{13.1}$$

This is not the same "no-arbitrage" condition that is relied on earlier to create the no-arbitrage condition between spot and forward (delivery) price. It is a no-arbitrage argument in the classical sense that selling options would depress their price, while simultaneously "buying the underlying" would bid-up its price (see also [8.a] and [8.c]). In this particular instance it is assumed that the price of the underlying is "sacrosanct", since it is the option's value that is being estimated, and so this no-arbitrage condition results with the conclusion that the (early exercise) option's price must be at least equal to the intrinsic value.

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358 This assumption is sometimes expressed as "utility maximising" or as part of the Rational Market Place Hypothesis. In reality, there are many exceptions in the markets to this due to a large number of psychological and "trading reality" factors. Economists have, strangely, highly idealised theories that attempt to account for this. Not surprisingly those theories are of little practical value. Trader's simply adjusted prices and bid/offers as appropriate for the circumstances, regardless of academic principles.

Notice that the previous no-arbitrage condition (i.e. between current price and delivery date forward price) is a necessary requirement for this "early exercise no-arbitrage" condition.

> Caveat: Since this early exercise trigger strategy assumes a very specific (no-arbitrage) relationship between the current underlying price and forward prices, it cannot possibly account for a large number of real world factors.
>
> For example, suppose that at the current moment the option's value is less than intrinsic, and so you trade out of the option into the underlying/intrinsic. It is demonstrable that, in the real world, the actual evolution of forward price, say, to what was the delivery date prior to early exercise, will result in a P&L that is worse compared to some other early exercise on future date. There exist many variations on this theme.
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> The point is that the instantaneous no-arbitrage relationship between spot and forward used for the early exercise trigger/argument is just that, instantaneous. A moment later, the spot and forward prices will be likely very different, though there too a spot-forward no-arbitrage "instantaneous" condition may be argued, but at a different price. This relates back to the considerations illustrated in Section 6.6 and 8.6.4.
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> In short, real world "early exercise trigger strategies" do not conform nicely to the idealised theoretical framework, and you will wish to consider addressing such issues as relevant to your trading and mandating.

The application of this "instantaneous dual no-arbitrage argument" is usually also accompanied by a "demonstration" that it is never optimal to exercise American calls early when the drift takes on special values. Notably, the most common presentation of this point is that early exercise of American calls is never optima if it is no-dividend paying, or no income yield option (i.e. $q = 0$, so drift = r, instead of r-q). Figure 13.1 – 1 illustrates a European call and put valued at one instant in time in relation to their intrinsic values (black lines), for three different drifts: Positive drift (red), zero-drift (green), and negative-drift (blue). As detailed in previous Chapters, varying drift has the effect of "shifting" the pay-out profiles up/down (as it effectively moves the forward price).

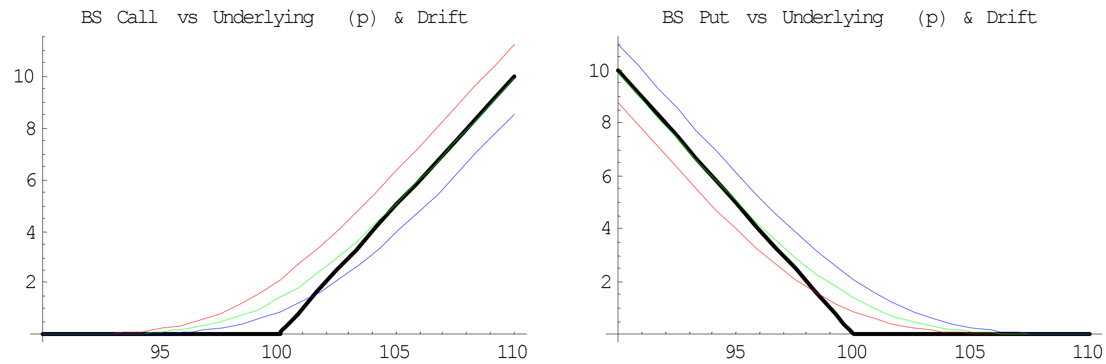
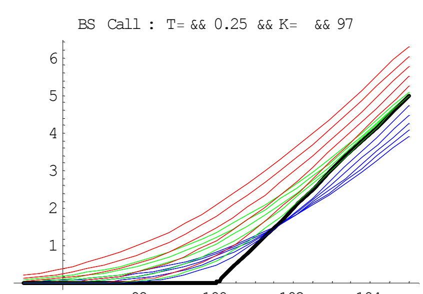
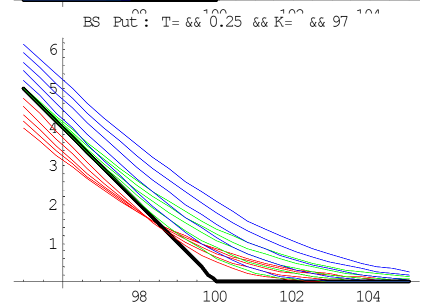


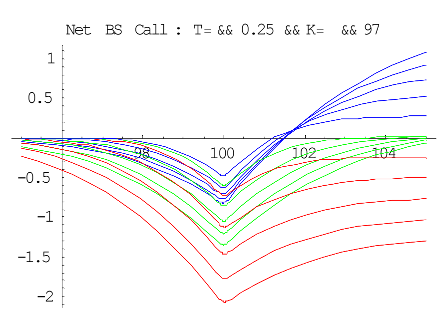
Figure 13.1– 1. European call (left) and put (right) comparing current value for three different drifts: positive (red), zero (green) and negative (blue), and intrinsic (black).

Crucially, negative drift for calls (blue)) and positive drift for puts (red) result with (European) option values below intrinsic for certain market conditions. These are conditions where the trigger strategy from above implies early exercise.





By contrast, the positive-drift case for calls (red) and negative-drift case for puts (blue) remain above intrinsic. Indeed, one may simultaneously vary also time to expiration and strike (i.e. moneyness) to show that this "drift-early exercise" property holds for a wide range of conditions. The images to the right show the above call result, but repeated for three different times to expiration and three different strikes. Clearly, for calls (upper) the positive drift cases (red) are always above intrinsic, so it is not every optimal to exercise early (for this trigger strategy) when drift is positive. By contrast, the puts exhibit the complementary effect, with negative drift (blue) always at or above intrinsic, suggesting that it is not optimal to exercise puts early when there is negative drift.



This exact same point can be considered by creating profiles that are the "net" difference between the option's values and the corresponding intrinsic values. The image to the right shows this for calls. As above, the positive-drift cases (red) are always less than or equal to zero, meaning it does not pay to exercise early (for the current trigger strategy).

A complementary conclusion for puts is almost true.
Indeed, the conclusion for the zero-drift case is not quite what it appears to be. In particular, recall the standard BSM environment is a geometric or returns based model but