market moves the "other way", the hedge loses money (the other two quadrants). Thus, on average, this hedge is not working since on average the P&L surface is "under water".

Sometimes "cheap & cheerful" methods can be quite useful in rectifying difficult position keeping problems. In this example, one "practical" solution is to estimate how much under water the net P&L on average[485], then price the EofA by adding that difference to the theoretical price. There are various other techniques to deal with such as issue, as detailed in [9.a].

16.1.7.7 Yield Curve Slope Option Example

Much of the discussion of the valuation of exotics reflects the "shoehorning" by "quants" to derive models within a BSM framework. This Section is provided to illustrate the valuation of yield curve SO assuming it to be a Quasi SO (i.e. Class 2), and so amenable to valuation via Margrabe's method, and a little insight into how traders "shoehorn" valuation issues into reality.

Suppose a client had reason to believe that The Fed would be enacting one of its "Twist" style Quantitative Easing (QE) operations by selling 10-year T-Notes against buying 30-year T-Bonds. If they did, the long end of the curve would "rotate" or "change slope" downward (i.e. invert, with 30-year yields dropping and 10-year yields rising). In IR "lingo" the expectation is for "flattening" (of the yield curve). Suppose further that the client is a fund manager wishing to hedge their existing bond portfolio against such a rotation. There are a variety of approaches with varying pros/cons as discussed here [8.b] and [9.a]. For present purposes, the fund manager decides to go with a 10's-30's yield curve spread option, and it is your job to price a suitable structure.

> Aside: There is much to discuss regarding yield spreads. This includes various considerations regarding the distribution of yields and their relationship to the distribution of returns, and then the relationship of all that to the distribution of prices, which are assumed to be Log-Normal in a BSM universe. This also raises considerations regarding term-structure methods, many of which are "arithmetic". These and other deeper points are deferred to [8.b] and [9.a].

> Aside: Regardless of the pros/cons of various finer technical issues relating to valuation models, the (IR) markets "love" Black76 for forwards based structures such as many IR derivatives. This means the most likely solution in the real world

[485] For example, at what P&L level would a perfectly flat horizontal slice have as much P&L "above" as "below". That is the "average" loss, and so that is the average amount that must be recouped somehow. Clearly, from a purely mathematical perspective, this approach does not provide a unique solution. Thus, heuristics are used as discussed in [9].

would involve the continued use of Black76, but altering the contract to permit sensible position keeping, or at least "acceptance" of the reliance on Black76, as introduced below, and detailed in [8.b].

Your solution is to price a SO based on yields and to rely (mostly) on the EofA formula for the reasons cited above, and in the previous Sections, but "adjusted" due to certain "real world concerns". In particular, you are concerned with two immediate issues:

- The market quotes IR vols in terms of "relative yield vol", as would be normal when using, say, Black76 to value swaption etc. However, when dealing with spreads, the (relative) yield vols of each of the legs in the spread are not "additive". To see that, ignoring correlation for a moment, consider a 10% yield vol on 10-year instrument currently yielding 5% in relation to a 30-year with yield vol of also 10%, currently yielding 7%. Thus, the 5-year rate would be in the range of +/-10% of its 5% for a 1 standard deviation range, and similarly the 30-year's 1 SD range would be +/-10% of 7%. Thus the change in the difference between them would be +/- 5 bps for the 10-year, and +/- 7 bps for the 30-year (for 1 SD). That is, a 10% relative change in the yields causes the yield spread to move +/-7 - +/- 5 bps, not 10% +/- 10%. Alternatively, suppose the market rallied and the 30's – 10's spread moved from the current 2% to 2.3%, i.e. the spread widened 30 bps. The change in the spread does not care what the actual individual yields are. Therefore, the spread's volatility is not relative to either of the yields, but is absolute. As a consequence, the correct accounting for the spread vol is to use absolute yield vols as the inputs[486], not relative yield vols[487].

Aside: Some care is required when speaking of yield volatility. Be sure to know whether it is quoted on an absolute[488] or relative basis. If it is quoted on a relative basis, then the vol is relative to the yield. In most cases, it is quoted on a relative basis, and then for the approach here, a conversion is required. For example, 13% (relative) vol on an IR instrument with a yield of 3% is an absolute yield vol of 0.13*.03 = 0.0039, or 39 bps.

Thus for this, or for other reasons considered here [8.b], the portfolio vol expression for yield spread used here is:

$$\sigma_{YSprd,Abs} = \sqrt{(y_1\sigma_1)^2 + (y_2\sigma_2)^2 - 2\rho_{1,2}(y_1\sigma_1)(y_2\sigma_2)} \qquad (16.48)$$

[486] In fact, real spread vols are a function of the size of the spread and in fact yield levels, but these aspects are considered in "fancier" models and trading in [8.b] and [9].

[487] A full derivation is available in [8.b].

[488] In some circles, absolute yield vol is referred to as "basis point" vol, since it is expressed as "bps".