This is the forward price "mountain range" that results from the simulation. The expected value on any of the forward dates (i.e. on any of the "cross sections") is the average of the histogram for that date.

The mountain range has the expected properties. It drifts upwards as required by the model, and it has Log-Normal distribution in prices (as the standard geometric forecast should).

<blockquote>
Caveat: although it is convenient to think of the MC simulation as a collection of paths, the actual MC valuation is the "cross sections". That is, the MC approximation of the integral solution of the DE is the average on any one forward date. Thus, there are various technical matters discussed in Section 19.4.4, and [3.a] that apply on a "forward date" basis, separate from any issues on a "path basis".

As an illustration, the random numbers used between dates must be (amongst other things) independent. That is, there must be a completely new and independent "bathtub" for each of the forward dates in the simulation, see [3.a].
</blockquote>

19.4.2   Pricing a Vanilla Option with Monte Carlo Simulation

The approximation of a vanilla call option by MC is nearly as easy as the examples above. Here too it is possible to perform a single time step or many time-step simulations. The single time step simulation is possible since a vanilla European call option does not (in of itself) require any intermediate calculations. Nevertheless, a multi-step MC will be used. There are a number of reasons for this; one of which is that the multi step MC permits later addition of features and calculations of interest (for example hedging rebalances with PaR).

The procedure is as before, with one extra component.

- Create the realisations (i.e. the paths of forward prices of the underlying)
- At end of each realisation apply "pay-out" function (this is the new bit)
- Value = PV of (weighted) average of trial pay-outs

Assume that the market is at 100, with a volatility of 15%, and you are required to price a European call option struck at 100 with expiration in 6 months, a funding rate of 10%, and a dividend rate of 6%.

Therefore, the pay-out function is *Max[P_N-100,0]*. This means, at the end of each path the calculation must compare "the final result (i.e. underlying forward price)" with the strike (in this case 100), and if that difference is positive, then keep it for later averaging, otherwise set it to zero.

Creation of a single Monte Carlo path – this begins exactly as before:

$$P_{new} = P_{old} \exp([r - \sigma^2/2] t + \sigma \sqrt{t} R)$$
$$P_1 = P_0 \exp(.0041 + .15 \sqrt{1/12} * \mathbf{0.39956}) \quad \text{or } P_1 = \mathbf{100} \ (1.0216) = \mathbf{102.1594}$$
$$P_2 = P_1 \exp(.0059 + .15 \sqrt{1/12} * \mathbf{-1.53436}) \quad \text{or } P_2 = \mathbf{102.1594(.9395)=95.9817}$$
$$....$$
$$P_6 = \underline{\mathbf{106.0613}}$$

However, this last value must be tested against the option's pay-out, and so:

==> **Pay-out** = Max[106.0613-100,0] = **6.0613**

The pay-out for several paths is shown in the last row of Figure 19.4 – 6, the penultimate row is the forecasted forward price for that path's expiration date.

Exp (days)	180	dt(days)	30	R	0.1
P0	100			D	0.06
Vol	15%			Days/yr	365

Days 2 Exp	Path 1	Path 2	Path 3	Path 4	Path 5
180	100.00	100.00	100.00	100.00	100.00
150	98.19	99.68	98.24	101.61	90.65
120	97.39	100.23	98.04	104.85	88.95
90	107.76	98.49	98.60	107.63	97.09
60	115.08	106.28	94.71	108.75	91.54
30	108.47	115.25	100.99	102.97	96.32
0	107.46	115.39	94.10	110.04	97.99
Option Val	**7.46**	**15.39**	**0.00**	**10.04**	**0.00**

Figure 19.4 – 6. Several MC paths of forward prices, and the option pay-out on the final forward date. The average of the last row is the "expected option" price on the forward date.

After 1,000 paths, the approximate option value is given as:

Opt $_{last}$ = **7.46** path 1
Opt $_{last}$ = **15.39** path 2
Opt $_{last}$ = **0.00** path 3

Ave = 5.22

Opt $_{last}$ = **10.04** path 1000

The PV of 5.22 is 5.064 (PV'd on the basis of the funding rate[518]), and closed form solution from Black-Scholes-Merton is 5.097 (and so is well within 3% confidence interval expected for 1,000 paths).

Congratulations, you have done your first MC **option**

A larger number of paths would have a result closer to the BSM result. The acceptance of the approximation error (and thus the number of paths) will depend on your specific circumstances. A risk manager worried about large-scale effects may accept a 3% error. A trader pricing an option in a liquid market must be able to have an approximation error that is less than the bid/offer spread.

There are specialised methods that permit error reduction without the need for an excessive number of paths, such as those highlighted in Section 19.4.4 and detailed in [3.a].

19.4.2.1   MC Code

The spreadsheet approach shown above is too cumbersome for practical application as it would require many thousands of columns, and the calculation speed is poor. "Proper" coding is required. As an illustration of an EDUCATIONAL implementation of a vanilla option MC code, consider the VBA listing below:

```
'          A very simple Monte Carlo Program for Vanilla Options
`          EDUCTIONAL USE ONLY - for illustrating the MC method
'
Function TG2QM_MC_XOpt_SimpleCall1 (PRICE As Double _
, Strike As Double _
, rate As Double _
, DIV As Double _
, YRS2EXP As Double _
, VOL As Double _
, NPATHS As Integer _
, NSTEP As Integer) As Double
'
Dim ISEED As Integer
Dim k As Integer
Dim i As Integer
'
Dim exprqt As Double
Dim exprt As Double
Dim dt As Double
Dim bb As Double
Dim cc As Double
```

---

[518] The PV is required since, amongst other things, the Black-Scholes result is on PV basis.