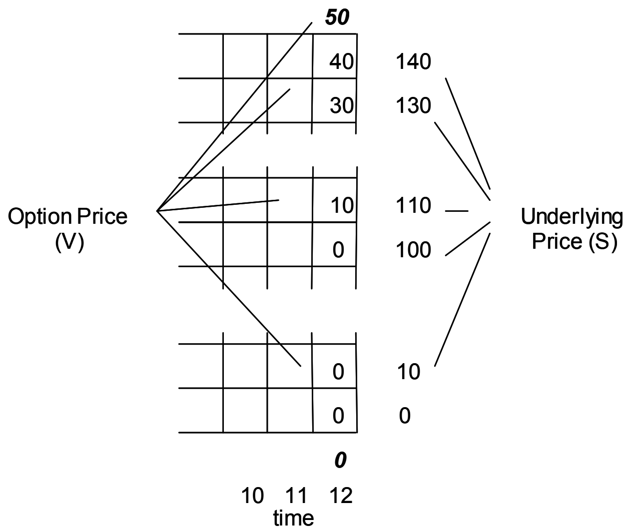

Figure 19.6 –5. Initial conditions and boundary conditions for initial conditions in an explicit FD.

19.6.3 Explicit Finite Difference Black–Scholes Formulation

The building blocks in the previous Sections can be used to construct an FD approximation for the Black-Scholes-Merton PDE. Applying the following approximation from (Equation (19.42)):

$$\frac{1}{2}\sigma^2 S^2 \frac{(V_{i+1} - 2V_i + V_{i-1})}{(\Delta S)^2}\Bigg|^{n-1} + rS\frac{(V_i - V_{i-1})}{\Delta S}\Bigg|^{n-1} - rV_i\Big|^{n-1} \cong -\frac{(V^n - V^{n-1})}{\Delta t}\Bigg|_i \qquad (19.43)$$

to the remainder of the mesh completes the valuation of the option. Rearranging this expression results with:

$$-\Delta t\,[aV_{i+1} + (-2a + b - r - \frac{1}{\Delta t})V_i + (a-b)V_{i-1}]\Big|^{n-1} = V^n\Big|_i \qquad (19.44)$$

where

$$a = \frac{1}{2}\frac{\sigma^2 S^2}{(\Delta S)^2}\Bigg|_i^{n-1} \text{ and } b = \frac{rS}{\Delta S}\Bigg|_i^{n-1}$$

Volatility	35.0%
Funding Rate	10.0%
dS	10
dt	0.021
Strike	100
DPY	365
Years to Exp	0.25

Using these expressions for option parameters as shown to the right, a spreadsheet calculation yields the results in Figure 19.6 –6. The table of numbers shows the option's value at various times and level of underlying prices. The "greyed" rows represent boundary conditions, while the shaded column is the *initial* condition defined by the pay-out formula at expiry. The value of the option today is obtained by looking at the first column on the left at the row with the current underlying price. So, if today's underlying price is 100, then the option value is 7.3.

Underlyin	Days to Expiry 7.6	15.2	22.8	30.4	38.0	45.6	53.2	60.8	68.4	76.0	83.6	91.3	
150	51	51	51	51	51	51	51	50	50	50	50	50	150
140	41.9	41.7	41.6	41.4	41.2	41.0	40.9	40.7	40.5	40.4	40.2	40	140
130	32.3	32.1	31.9	31.7	31.4	31.2	31.0	30.8	30.6	30.4	30.2	30	130
120	23.0	22.7	22.4	22.0	21.7	21.5	21.2	20.9	20.6	20.4	20.2	20	120
110	14.3	13.9	13.5	13.1	12.7	12.3	11.8	11.4	11.0	10.6	10.2	10	110
100	7.3	6.9	6.4	6.0	5.5	5.0	4.4	3.8	3.1	2.3	1.3	0	100
90	2.7	2.4	2.1	1.8	1.5	1.2	0.9	0.6	0.3	0.1	0.0	0	90
80	0.6	0.5	0.4	0.3	0.2	0.1	0.1	0.0	0.0	0.0	0.0	0	80
70	0.1	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	70
60	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	60
50	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	50
40	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	40
30	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	30
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	20
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	10
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0	0
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	

Figure 19.6 –6 a) Spreadsheet implementation and solution of explicit FD option pricing

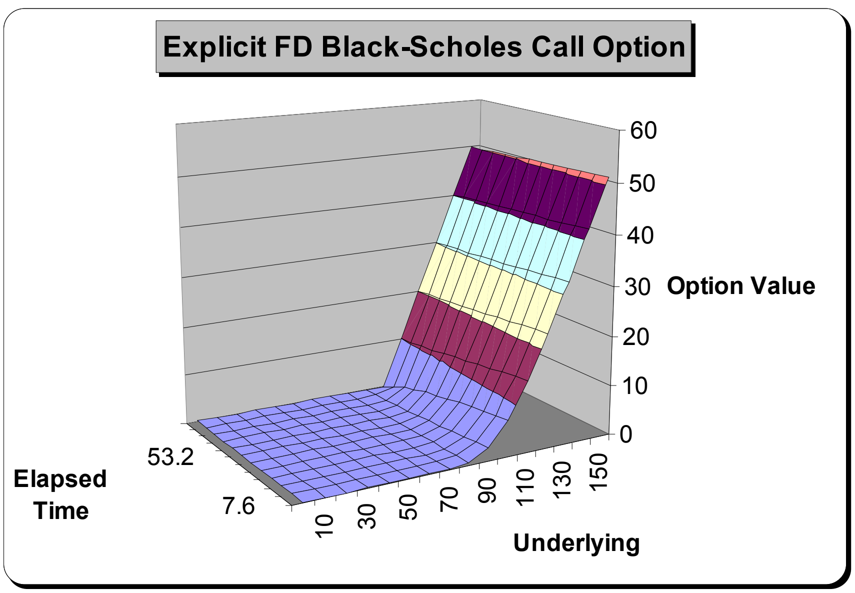

Figure 19.6 –6 b) Plot of explicit FD solution

This approach produces the option's value for every "node" in the simulation space, rather like the Tree method's backward induction pass. In fact, it can be shown that one particular implementation of the explicit FD procedure is identical to a "Trinomial" Tree implementation. However, there is one (and only one) specific FD construction that provides that. The Tree methods do not have the freedom to set the PDE boundary conditions, while the FD does, and so the FD method can solves "other" problems just by virtue of altering the BCs (see [3.b] for details).