require a 3-dimensional representation (e.g. no longer a surface, but rather a "volume", and even more dimensions/factor would require a "hyperspace").

20.3.2 Volatility Skew

Volatility skew, sometimes called the volatility "smile", is the relationship between (implied) volatility and option strike. As always, this relationship reflects also supply/demand conditions since (quoted/traded) implied volatilities are a proxy for prices. However, it is possible to extract important information from this relationship for directional and arbitrage purposes, and of course also to use the information to interpolate data for contract "specs" not quoted, but which need to be priced, hedged, reported, etc.

Here, the word "skew" is not used in the same way as it is in the context of the third moment of a statistical distribution. Rather, it is simply that this has become the common "name" applied to this type of plot, where away-from-the-money options are quoted at different (usually higher) implied vols compared with the ATM vols.

Figure 20.3 – 7 illustrates a "vol skew" plot. Again, keep in mind that only the data at the "points" exists, and the rest is the result of interpolation (which is subject to fitting assumptions). In this particular example, the skew is "positive" to both sides of the ATM option (i.e. the strike closest to the current price of the underlying), and as such, it is suggestive of a "smile".

Figure 20.3 – 7. Volatility skew ("smile") for (the same) options with different strikes.

Why should the implied volatility away from ATM be greater than the ATM implied vol? In other words, why is the ATM option (relatively speaking) "cheaper" than the other options. The classic explanation of this is to say "*if the market actually moves to the strikes* *that far away from the current prices, then something important must have increased volatility, and the market makers will charge for it now since they will incur increasing hedging slippage if that actually happens*".

This skew is nowhere in the standard theoretical models. Thus, once again trading requires real world adjustment/management for model vs. real world differences.

It is common to plot a single line for volatility skew, but in fact it is two separate lines merged together. Some publications only show single line, where the upper (right-side) portion of the skew is the upper part of vol skew from call options[569], and the lower (left-side) portion of the skew is the lower portion of the put curve.

The shape of the "smile" is not always a "smile". Sometimes it's "asymmetric" or even a "frown". Figure 20.3 – 8 illustrates the vol skew for S&P options. The S&P used to exhibit a "smile" for extended periods. Since the crash of Oct 1987, however, market makers have been "charging extra" more often for puts since the S&P's sometimes "saw tooth" (see Section 20.7) behaviour can be ruinously expensive for sellers of options (particular for those relying on Delta position keeping strategies), as there is no easy way to hedge that risk (see also Chapter 22).

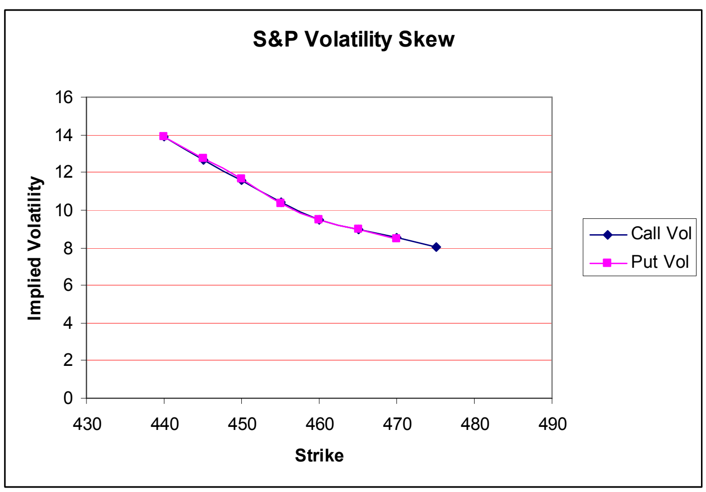

Figure 20.3 – 8. Typical volatility skew for the S&P following the Oct 1987 crash.

Figure 20.3 – 9 shows the volatility skew with a "kink". This type of effect can occur due to supply/demand conditions (e.g. there is a "one-way" market since market makers are trying to cover their barrier options position such as per the example discussion in Section

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[569] A little care is required with the terminology since IR products are often spoken of in terms of their yield (i.e. in terms of interest rates) and since prices and rates are inversely proportional, a call on prices is a put on rates and so forth. For example, a receiver swaption is a call on the IRR of the underlying forward swap, but it is also a put on the price of the equivalent or "synthetic bond".

20.2.1). It can also indicate a possible arbitrage condition[570]. The specifics of this type of arbitrage condition are detailed in [8.a] and [8.c]. For the moment consider that it is similar to the "butterfly" trade on a "kinked" yield curve as detailed in [4] and [4.a]. Essentially the "arb" is this: Buying options on either side of the kink implies an option with a strike at the kink. Thus, the price of the in-between strike should follow a "smooth" or "monotonic" relationship due to this "equivalence". Much like a spread trade is used to replicate the "in-between" implied contract. Then, the arb would be complete by selling the "hump" strike option against the two long straddling strikes. As demonstrated in [8.c], this trade is (most of the time) actually an arb: if it were only possible. In reality, these types of humps may arise only on the bid or offer side due to special supply/demand circumstances. Put differently, the reason the hump exists is that somebody is buying a lot of those strikes from the market makers. If you tried to sell the "hump strike" to the market makers, they would not buy at the "hump price", but rather at a much lower price, thereby defeating the arb. Did you remember the definition of an arb: amongst other things, it requires liquid two-way flows, but usually only one-way flows exist in cases such as this example.

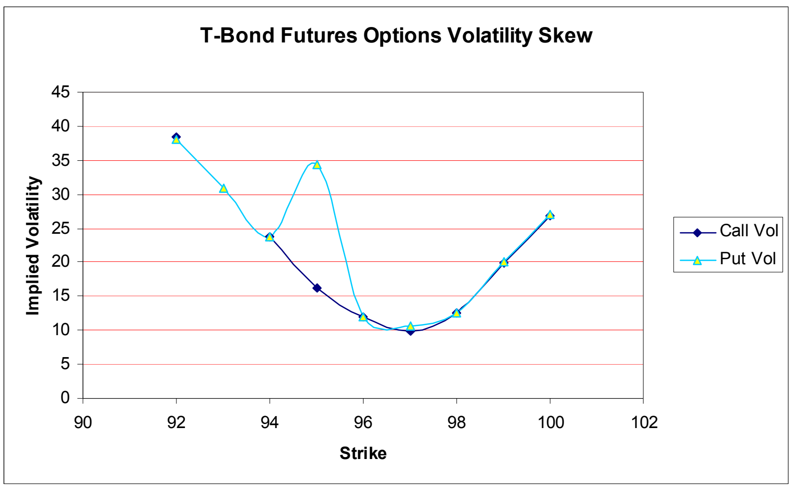

Figure 20.3 – 9. A "kinked" volatility skew.

As before, it is possible to combine vol skews with other important volatility factors, such as expiration, maturity of underlying, and so forth. This necessarily means surfaces, volumes, or in general "hyperspace". Again, some care (or even quite a lot of care) is required for the treatment of market data and especially in a multi-dimensional setting.

Notice that the existence of a vol skew "implies" that the BSM risk-neutral framework (i.e. that there is just one "mountain range "underlying the valuation formulas) is no longer supportable. However, some analysis is required to decide the size of the P&L discrepancy this would introduce, and if that discrepancy is material to your business.

[570] In fact, this type of trading, even for a discrepancy of a few ticks, is reported to be the "arb" that made a success of companies such as CRT and O'Conner in the 1980's.