

## 21 Profit-at-Risk (PaR): Optimal Risk-Adjusted P&L

Regardless of which part of the business you are in, holding period risk-adjusted returns (or P&L) analysis is the cornerstone of our business. Holding period P&L analysis embodies every aspect of trading, sales, structuring, risk management, and business management. It covers the entire “trading cycle” as introduced above and described in [1] or [2]: trade idea generation, structuring/execution/sales, position keeping, business management, and audit<sup>607</sup>.

Holding period risk-adjusted P&L considerations are especially important in options trading where not only is there a greater reliance on dynamic rebalance strategies, but also all valuation and risk/reporting systems implicitly assume a (very particular) dynamic rebalance strategy as their basis, and one that is not as realistic as one might like.

Consider that:

- Hedging/position-keeping strategy selection is best accomplished by choosing the rebalancing process that provides the best Risk-adjusted Returns (RaR's). Notice that this also immediately addresses the issue of “synthetic replication” efficiency, and may also assist in calculating the precise degree to which the position is “not risk free”, and thus how much to adjust the operation's (the trade's) RaR requirements.
- Selling securities and derivatives to clients would be much more efficient if a PaR analysis would show the client their holding period RaR with and without additional securities and derivatives under various market conditions.
- Directional trading strategy selection and testing of the risk and profitability of directional trading can be best accomplished by assessing the holding period RaR for a variety of market scenarios and trading rules.
- Arbitrage trading strategy is, from an analysis perspective, a kind of combination of the directional strategy analysis with hedging strategies with which to extract low- or risk-free profits, so clearly a candidate for holding period P&L analysis.
- Structuring analysis, much like the sales analysis, may well benefit by comparing the holding period RaR's for different types of structures and the construction of comparable structures by more than one approach.

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<sup>607</sup> Some elements of this Chapter anticipate the reader to have market experience. If you are new to the business, as usual, you may wish concentrate only on the main and emphasised points – at the least the first time through. However, try to have a go at understanding the fiddly bits anyway, since they do represent actual and real market effects and idiosyncrasies that simply do not arise in theory (and it is what separates the “wheat from the chaff”).

- Model arbitrage is similar to arbitrage analysis, with the focus being on the comparison of the RaR's of different valuation models, once again, a prime candidate for holding period P&L analysis. Model arbitrage analysis is included as a subset when holding period model validation analysis is performed (e.g. "does Black-Scholes-Merton work sufficiently well for our business mandate, and how much better would we be off moving to, say, term-structure models?" etc).
- Risk management implications for a desk or the firm for a variety of market scenarios, while including the firm's risk strategy (e.g. that there should be inter-period rebalancing – possibly of varying effectiveness), is clearly an ideal problem for PaR analysis (and notice how much more relevant this approach is, compared to VaR).
- Business strategy analysis at the desk, trading floor, or firm wide level: holding period P&L basis provides not only assessment of the risk implications, but also business management implications from a budgeting/capital allocation perspective, and comparison of "how well the firm's capital" is being used (compared to other possibilities within and outside of the firm).
- Bonusing and compensation policy may be driven by analysing the effectiveness with which a desk or trader employs capital and risk resources, or their position keeping efficiency (e.g. how much of the bid/offer spread did we keep vs. how much could we have kept?).
- Perhaps you are the Treasurer of a large corporation that uses securities and derivatives to manage various exposures, loans, and the like. However, you cannot afford a staff of professional traders, and so you may imagine that you are at the mercy of the investment bankers for pricing and analysis. Then a holding period P&L analysis calculator might be an inexpensive tool to help provide the type of insight<sup>608</sup> that a trading team might offer.
- A risk manager or senior manager may wish to show regulators (e.g. BOE/FSA, SEC etc) that the firm's hedging strategy is very effective and so should qualify for reduced capital requirements (possibly under CAD II etc). Clearly, they will be better able to illustrate their case and convince the authorities with a holding period P&L analysis that accounts for the rebalance/hedge effects, market dynamics, and so forth.
- Holding period analysis is also a primary tool for deciding the "prove it!" challenge. That is, when a claim is made about a valuation model, strategy, or "perspective", how is the case for that choice to be made?
- ..... etc

One particular implementation of such holding period analysis is the Profit at Risk (PaR) methodology introduced in this Chapter.

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<sup>608</sup> It is unlikely that (any time in the near future) there will be software that can completely replace experienced traders. However, a good PaR calculator would incorporate much real trading experience and real market idiosyncrasies, and thus at least provide a very good first impression of how "a case of bonds" or some "Asian options" would assist the treasurer's position. An even better PaR calculator should be able assist the treasurer in selecting the "treasurer's best" strategy.

This Chapter provides an introduction to the PaR methodology which received a detailed introduction in [1], a detailed technical/implementation treatment in [3.a], and detailed treatment in [11]. PaR is also used extensively in the “trading volumes” of this Series to demonstrate strategies and P&L issues. The accompanying software includes an “educational” version of *ART’s Pr/rO @ XL lite* calculator.

The essence of PaR analysis is, roughly speaking, in two parts. First, produce many simulations of the trading process covering the holding period of interest by varying any “variables of interest” (e.g. M2M prices), including sufficiently real world effects, such as rebalancing strategies, etc.. Second, analyse that collection of results to determine the risk-adjusted P&L of individual simulations and the collection of simulations to determine which of those results indicate the “best” risk-adjusted returns for your mandate by “auditing” the (simulated) trading process.

The first part of PaR can be performed in one of two slightly different ways, each of which provides slightly different types of information. These two variations are forward-testing, and back-testing. Forward-testing essentially creates parameters based on the “views” of the user, such as forecasts of prices and volatilities. Back-testing relies on historical data for such parameters.

Regardless whether important parameter originated from forecasted values or historical values, the machine must then be able to perform simulations of entire trading sequences over the holding period, which means that the machine must also be able to:

- Book deals.
- Know the rebalance rules.
- Rebalance or “trade” according to the rules.
- Track reality impact such as transactions costs, liquidity, etc.
- Report P&L’s, create risk reports, etc.

That is, the simulator must be able to “do” whatever it is that “you do” and your dealing systems etc do while trading over some holding period; at least sufficiently well to produce P&L’s that are sufficiently realistic and consistent with your or real world trading.

Thus, as advertised, the PaR methodology incorporates all of steps of “trading cycle” introduced in [1], and detailed in [2]. The “trading cycle” being the collection of steps of trade idea generation, structuring/selling, valuation, position keeping, and audit.

PaR analysis can be applied to single trades/structures, or entire portfolios and “big picture” issues. Most of the discussions here will be in the context of single trades/structures. Some discussion of the many extra layers of minutia relating to proper portfolio PaR analysis is provided, but most deferred to [11], and [14].

Remember, as with all tools (and PaR is just a tool, albeit a very fancy tool), be sure that investing time and money in such a resource is consistent with your business mandates. Buying or building a PaR calculator, especially a reasonably general one, can become quite expensive quite quickly. This means that, for example, a “discount broker” who is providing execution only may not be able to justify (or even “rationalise” 😊) the cost/benefit ratio of such a device. On the other hand, a market maker, senior manager, or structurers/sellers of complex products may wonder how they ever got along without one.

The word “scenario” arises quite a bit in this Chapter and in PaR in general. Some care is required since there is a “hierarchy of scenarios” as shall be seen. For example, guessing several forward prices generates those “scenarios”. However, those same scenarios of forward prices might be used to assess the difference between a vanilla option payout and a digital option payout, so here there are two (higher) scenarios (the options) relying on many (lower) scenarios (the forward prices), and so lead to further “levels” in the scenario hierarchy.

Finally, almost everything in “PaR-land” is a portfolio problem. Though there has been some minor introduction to portfolio issues, this is necessarily a complex topic, especially in a real world setting. This Chapter does provide some insight into some reality impact issues that only arise in a portfolio context, but a thorough treatment of portfolio effects is left to [11], and to a lesser extent to the TG2 “trading” books.

Caveat: The essence of PaR analysis is a “machine” that has sufficient knowledge of trading and the markets. It is absolutely crucial the sufficient reality is built into the analysis. Quants can produce beautiful looking simulators, but without a deep understanding of the markets it is, as usual, “garbage in/garbage”. Only here it is “beautiful garbage”. Indeed, it can be so “beautiful” that garbage becomes believable.

If you cannot be sure that a sufficient degree of reality is built into you PaR results, then “run away” as fast as you can. Contact [ARTWare@Arbitrage-Trading.com](mailto:ARTWare@Arbitrage-Trading.com) for more information.

## 21.1 Introduction to the PaR Methodology

Profit at Risk (PaR) aims to capture all important trading and P&L effects over a holding period, and then provide analysis of the risk-adjusted performance of the events/strategies contributing to that process. Generally, the analysis will also address the question of “best” or “optimal” choice among the strategies tested. This Section considers what actually happens in the “real world”, and then constructs (at a conceptual level) the components that a PaR methodology would require. The Sections following this one provide more detail on each of those main PaR components.

### 21.1.1 Trading, Hedging, Selling, Managing, and Real World P&L

What actually happens during the “holding period”? Figure 21.1 – 1 provides a schematic illustration of the process.

- On some start date, you or your clients establish a position. It may be accompanied by any necessary supporting trades to ensure compliance with capital, credit, and risk limits. The “portfolio” is marked-to-market (M2M'd), and there will be some funding effects, transactions cost effects, etc. All of these must be reported or accounted.
- Then, the position keeping or investment process begins. Each “interval” or time step thereafter must be managed<sup>609</sup>.
- The typical “management” of an interval will include risk assessment/reporting, P&L reporting, funding, etc, and any rebalancing that may be required for the preservation of the strategy or limits. These “activities” imply P&L effects.
- This process continues to the end of the holding period (this could be any desired “overall” interval, such as expiration of the trade, or an interval for the annual report, etc). At that time, the net P&L of the process may be calculated by comparing the M2M of the target position to the “sum of the P&L effects” resulting from the (in this case Daily) “activities”. Notice, that this net P&L calculation may also include capital utilisation, hedge slippage, maximum or average draw-own, and many other important measures.
- This net-P&L result can now also be converted to holding period returns, and may be based on either or both of capital- or risk-adjusted basis (e.g. RoE, RoA, etc, and with/without indexing).

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<sup>609</sup> The interval could be anything from “tick” to annual, etc, though for the moment it maybe most convenient to think of this as a Daily frequency.

The sum total of this sequence of events (i.e. the “process”) is exactly what a trader does, or what the buyer (client) of securities/derivative experiences.

It naturally follows that in an ideal (but realistic) world, all trading, selling, and management decisions would be considered in terms of their impact on this process.

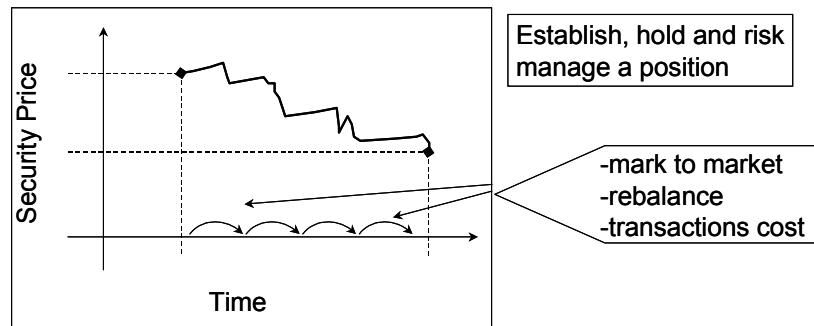


Figure 21.1 – 1 The holding period “process” for executing, managing, and closing a trade.

## 21.1.2 Trading, Hedging, Selling, Managing, and Simulated P&L

One approach to assessing trading, selling, and management issues in a manner that captures the effects on the holding period RaR, is simply to simulate the entire process.

There are many important considerations to such a simulation. One important question is that of determining the mark-to-market prices (or at least the prices used in the risk calculations) over the holding period simulation. For example, if the simulation is for a 3-month option with a Daily reporting/limit checking frequency, then you must supply “prices” for each “day” of the simulation. These “simulation” prices may be generated by models expressed by numerical methods (e.g. Monte Carlo simulation), or they may be “drawn” from a database of historical prices. Usually, the numerically generated prices are “forward looking” (e.g. a guess about the future), while the historical data is necessarily backward looking. This leads to forward testing and backward testing.

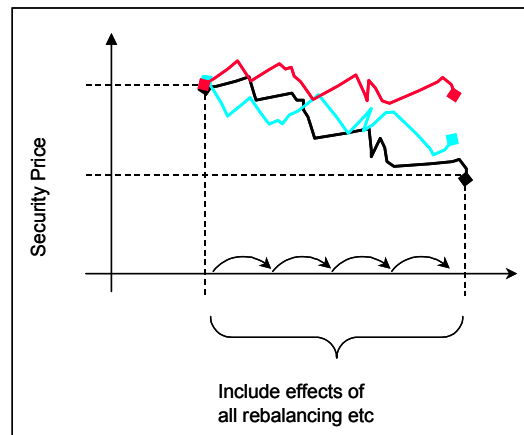


Figure 21.1 – 2. The payout profile of a bond position showing the both the sensitivity of the