22.12 Directional Volatility Trading vs. Volatility/Options Arbitrage

As repeated *ad nauseam* throughout this Series, the word "arbitrage" is perhaps the most abused word in the business. Even on the rare occasions when a (more or less) "proper" arb exists, there will be (almost surely) some risks. This is particularly true of options where (almost surely) arbs will require an entire sequence of very specific rebalances. Since there is no guarantee that any such rebalance may be possible on a future date, there is no guarantee of arbs (remember all "proper" arbs require liquid two-way flows). It is fairly common for traders to refer to some directional trades as "arbs", even when it is tacitly understood that it is in fact a directional trade with directional risk/return.

Proper arbs should generate "excess returns" with little or no risk (e.g. little or no P&L variability) regardless of market conditions. Thus, in a PaR/EF sense, the average P&L should always be positive, and the P&L variability should be "vanishing".

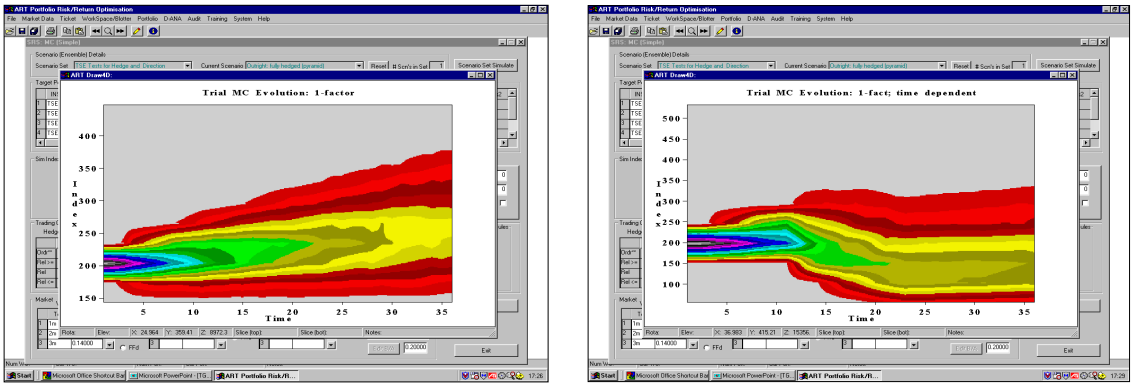
With options, particular when a relatively high very specific rebalance strategy is required, even small changes in conditions/rebalances may transform the trade from an arb to a directional trade.

As such, the opportunity to take inappropriate risk either due to "honest mistakes" (e.g. incompetence), or due to deliberate misrepresentation of facts or abusing limits/authorities, is very high with "options arbs".

The long and detailed considerations involving options arbs begin in [2], [8.c], and [14], and in many cases will require proprietary treatment not found in any books.

Perhaps the most common variety of option arb is "model arb". For example, the standard model produces a particular premium, while another (e.g. fancy) model produces a different premium. In these cases, the arb profits (if they exists) are extracted by "paying/receiving" the standard price, but synthetically replicating via rebalances driven by the "fancy" model's "triggers". These triggers need not be restricted to Greeks, as discussed below.

As an illustrative example, consider a slightly concocted situation which assumes only two types of market regimen. One is the usual/standard drifting/constant vol case, and the other is the "(price & vol) saw tooth" case. Figure 22.12 – 1 illustrates the "mountain range" of forward prices for these two types of market conditions.



a) "Standard" case. b) "Saw tooth" case.

Figure 22.12 – 1 a) A "flame thrower" image of MC forecasted forward prices represented as a contour diagram of the forward date histograms using the traditional forecasting model. b) A "flame thrower" image produced using a "time dependent" forecasting model allowing for drift and volatility to vary over the forecasting horizon: in this case representing a future that has a sudden sell off with increased volatility, and then a gradual return to "normal" conditions: that is, a price/vol "saw tooth" scenario.

If only these two market conditions exist, and since the standard model is (at best) only consistent with the "standard" market conditions, one may imagine the possibility that rebalances/synthetic replication under "saw tooth" conditions may over/under-perform the standard model. Would that lead to an arb?

Figure 22.12 – 2 shows the results for the usual Delta/Delta rebalance choices for the "saw tooth" case (blue), plotted together with the two "vol directional" cases for 15% and 18% vol. The saw tooth forwards for this case were generated by using +6% drift and 15% vol for the market/prices evolution for the first 120 days, then a –7% drift and 33% vol (i.e. large sell off) for the second 120 days, and then the market returning to "normality" with a +6% drift and 17% vol for the final 120 days of the 360-day par option's life.

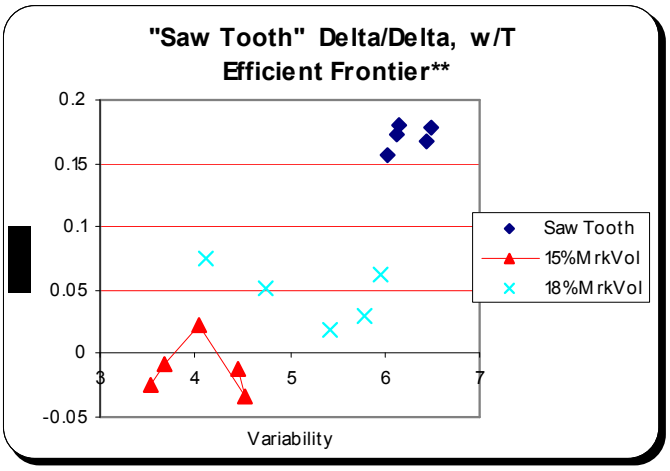


Figure 22.12 – 2. EF results for a 360-day par call with 0.1%, 10%, 20%, 30%, and 50% Delta rebalance limits for a "saw tooth" market scenario with 6% drift/15% vol in first third, -7% drife/33% vol during the second third, and 6% drft/17% vol for the final third of the holding period (c.f. BSM assumed 6% drift/15% vol).

Clearly, the strategy performs extremely well during the saw tooth regimen (blue). Is this an arb? Strictly speaking, no! This is just a very lucky directional trade, as emphasised by the substantial P&L variability.

However, it may have the "beginnings" of arb. For example, notice that although the profitability of the saw tooth case is considerably higher compared to, say, the 18% "standard" case, the saw tooth's P&L risk is only "somewhat" higher. Thus, if, for example, the saw tooth's P&L variability could be reduced (even if its P&L remains the same), then on an EF basis, it may very qualify as a "quasi-" or "risk-arb[666]".

A "quasi-arb" in this context may exist if one strategy had a higher P&L for the same P&L risk as another strategy, since then one could "arb the P&L risk" (at least on average). As considered in [8.c], the possibility of "quasi-arbs" vs. "full proper" arbs introduces many new policy risk/return considerations.

Moreover, there may be variations that fine tune the strategy towards arb-like risk/return characteristics. For example, if the only two regimens that exist are the "standard" and "saw tooth", and then based on the EF's in the Figure, the worst case scenario would be the "peak" red result, since one could set the rebalance frequency in the following way:

- If the market vol is equal to (or sufficiently close to) the inception date vol (here 15%), then apply the (peak) 20% Delta rule.

- If the market is above the inception date vol (perhaps by a suitable margin), then use the "best" Delta frequency from the saw tooth (blue) set.

- If the market vol is below inception date vol, then use the "best" Delta frequency from a suitable analysis (not shown here, but it would be similar to the blue results in Figure 22.10 – 1).

Notice that this "rebalance rule set" is triggered not only by "risk factors", but also by "market factors" (e.g. is market vol > inception vol, if yes, then use rebal Q, etc.). The rebalance rules could include a variety of conditions/triggers determined from (usually many) PaR analyses, and usually involving (many) PaR "back testing" simulations as illustrated in Section 22.14.3. Indeed, while (forward) PaR analysis is useful, it is just a fancy version of scenario analysis based on expectations. "Real" proof of the strategy requires at the very least PaR back testing, and often also test trades/trade audits.

[666] Here, "risk-arb" is used in the specific context of options arbitrage, as detailed in [8.c], not in the sense that some equity traders use the expression for WI's.