Remember, though, that this is a multi-dimensional problem, that the graph in Figure 22.2 – 2 is not exactly "straight", and that the three lines have not exactly converged is an indication that there still exists residual (non-linear) risk. Although the lengthy and detailed discussion of these residuals is left to [8.a], consider that there is at least the following to investigate:

- The hedging options have different term than the target option, and so as time moves forward, there will be an increasingly greater discrepancy in the hedge due to the divergence in the respective Theta profiles.
- That there is a difference in the terms of the options also gives rise to "vol curve rotation risk" and possibly other term structure effects/risks.
- Though the position may not now require very high frequency rebalancing, it may still need one or more later adjustments.
- The expectation for P&L performance (which still only apply "on-average") now may not be equated to the risk free rate (or any real world proxy), and the prudent manager would need to recalculate the risk/return requirements for the business (as compared to that proposed by the risk-neutral methodology).

22.4.2 PaR Synthetic Replication: Delta/Gamma, Pyramid Triggers

This Section applies a Pyramid-like strategy via PaR simulations to examine the trading performance (holding period risk-adjusted P&L) via the usual EF analysis. Due to the limitations of this Chapter, only three "big picture" scenario-sets are considered. Namely, each "family" of tests is based on choosing only one hedging option but varying its strike on the basis of 5%, 10%, and 15% moneyness. That is, each scenario-set will be hedged with either a 105, 110, or 110 strike hedging option of the same expiration as the target option.

This is a fairly serious constraint since in the real world there would usually be many options available for rebalancing, and sometimes rolling shorter dated hedging options provides a preferable risk/return character.

Also of critical importance, as previously, is the actual size of the transactions costs. Here, the matter is even more complicated since there are transactions costs for both the hedging option, and for the underlying (for Delta hedging). Often, the transactions costs for options are greater in terms of bid/offer spread compared to that of the underlying (though often less hedging option notional trading is required). As such, the results can be materially impacted by such differences and levels in transactions costs. For simplicity, here the transactions are assumed to be 10 bps on notional for both the underlying and hedging option, which may considerably overstate transactions costs for many real world setting.

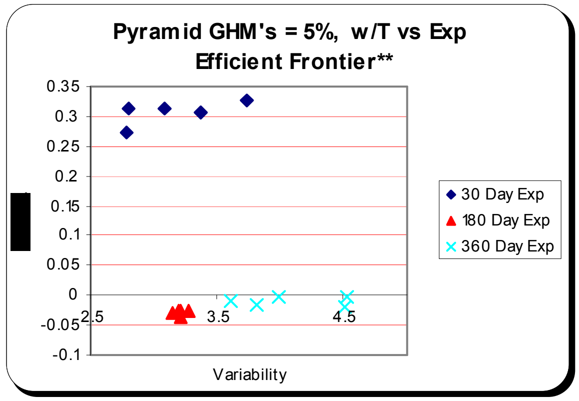
In addition, and as previously, the funding streams are based on the option's funding rate and dividend yield as appropriate; whereas in the real world funding would be based on some overnight and/or repo considerations. This too can have a material impact on the EF results.

> Caveat: Notice the difference between Pyramid vs. (general) Gamma/Delta strategies. With Pyramid, both Gamma and Delta are rebalanced when ever a "Pyramid Trigger" occurs (which may be set to anything, but here it is when either Delta or Gamma exceed some limit). By comparison, a (general) Gamma/Delta strategy could rebalance either Gamma and/or Delta when a suitable trigger occurs, and does not need to follow the "all Gamma first, and then remaining Delta" Pyramid rule. Thus, while (pure) Pyramid will always rebalance BOTH on a "trigger point", Gamma/Delta may only rebalance one or the other (or both), thereby leading to a different rebalance strategy compared to Pyramid. Yet again, this emphasises the "many permutations/combinations" issue.

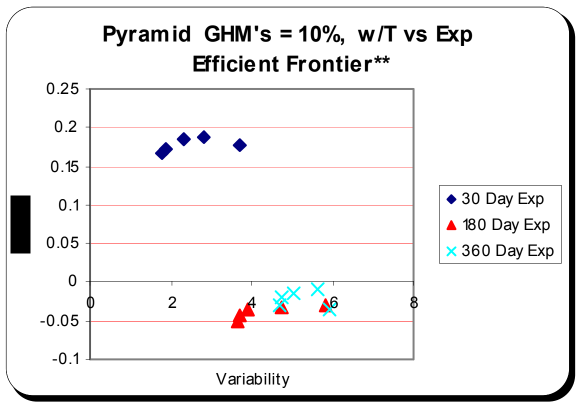
> Caveat: There are also numerous "technical complications" that otherwise require very lengthy discussion, as in [8.c]. For example, calculating the Gamma hedge rebalance ratio will be wrt the target/net Gamma. However, Gamma becomes rather "unstable" near expiration and near strikes, where it can range from large to small values. Thus, blind application of Gamma ratios can result in huge and unrealistic Gamma rebalances. These matters are ignored here, and the illustrations rely on rebalances that are mostly devoid of such issues, but then also necessarily of a restricted application. Of course, in the real world, these matters do not arise since traders will apply a "sensible" selection of options for Gamma rebalances and as appropriate.

Figure 22.4 – 3 a) – c) illustrate the three scenario sets with the 5%, 10%, or 15% Gamma Hedge Moneyness (GHM's) equating to 105, 110, or 115 strike hedging options. The usual three options (30-day, 180-day, and 360-day) are then tested for the usual variation of Delta rebalances with Delta limits of 0.1%, 10%, 20%, 30%, and 50%.

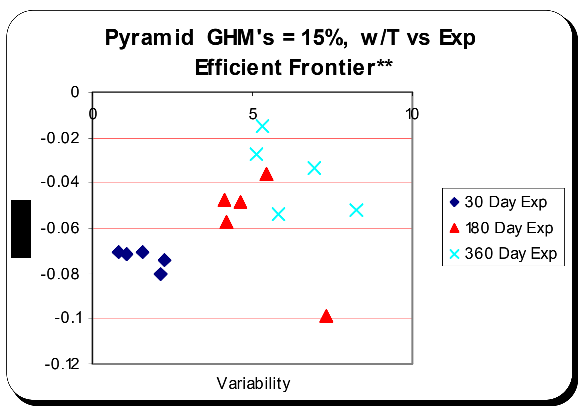
The low GHM scenarios (5% and 10% in Figures a) and b)) show a different character in that the 30-day option's performance is "out of line" with that of the longer dated option. Once the GHM moves sufficiently high (e.g. 15% in Figure c)), the character appears (more or less) "consistent" in the "big picture commensurate risk/return sense".



a) 5% GHM (i.e. 105 strike hedging option).



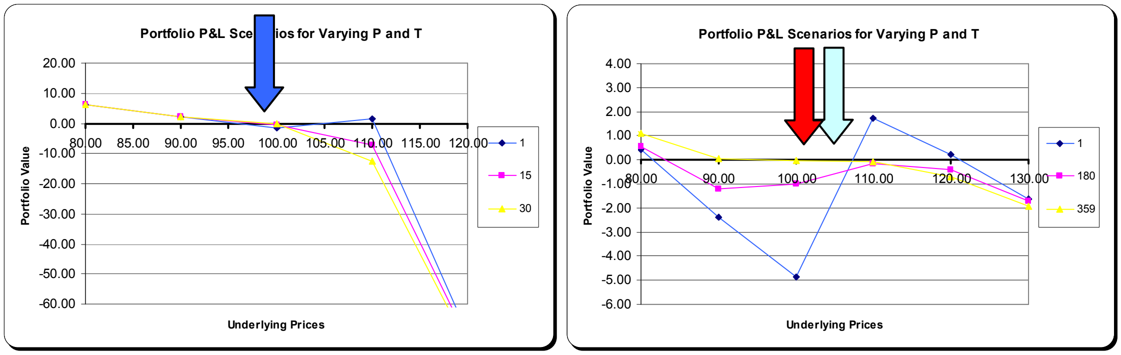
b) 10% GHM (i.e. 110 strike hedging option).



c) 15% GHM (i.e. 115 strike hedging option).

Figure 22.4 – 3. Scenario EF's for Pyramid hedging with 20% Gamma limit and varying Delta limit (.1%, 10%, 20%, 30%, and 50%) strategies for a par call Gamma hedged with 105, 110, or 115 calls referred to as 5%, 10% and 15% Gamma Hedge Moneyness (GHM's).

To understand why these EF's have these characteristics the main points to consider are the "position profile" and the expected forward price on expiration.



a) 30-day Option b) 360-day Option

Figure 22.4 – 4. a) Position profile after Gamma/Delta hedging with 110 30-day call option (i.e. 10% GHM, and b) after Gamma/Delta hedging of 360- day call option. The blue arrow indicates the location of the expected forward price after 30 days; the red after 180 days, and the aqua arrow after 360 days.

Figure 22.4 – 4 a) and b) illustrate the position profiles for the "net" portfolio at a point very near inception (yellow lines), about half way to expiration (pink lines), and just about at expiration (blue lines) for the 30-day, and 360 day options scenarios for 10% GHM's.

The 30-day's profile looks like a short call position since hedging a par call with a 110 call on a Gamma basis requires shorting almost exactly 8 contracts of the 110 calls for these short dated options. Thus, the character of the (8) 110 calls dominates the profile. The blue arrow shows the approximate location of the forward price on expiration date. It can be seen that, effectively, this strategy produces a short position in the 110 calls, and since the forward is well below the 110 strike, the short 110 call ends up "OTM" almost all the time (and since it is short, OTM "makes money").

This would seem ideal, but of course if market volatility was greater than expected, then the short 110 calls would generate a large loss. Conversely, as the market moved and wobbled around higher prices, the rebalancing size and frequency would increase, and then again alter the risk/return characteristics of this scenario.

By comparison, the longer dated options have "inception date" position profiles more like that in Figure b). Here, the "correct" inception date Gamma hedge ratio is about -0.9 contracts of the 110 calls. If only the Gamma hedge was applied, then the profile would look like that in the image to the right. That is, the net position is essentially a call-spread. Then, completing the Pyramid strategy by Delta hedging the residual Delta risk produces the "clockwise rotated" version of the profile seen in Figure 22.4 – 4 b).