interest rate on their funding). That is, their higher price is not arbitragible since it is only higher to account for higher costs.

This raises the question of "how would higher cost operations compete?" since their higher costs will necessarily mean lower average returns to the shareholders/investors. In reality, these calculations do not figure into the considerations of managers, though they should. At the very least, firms that have higher costs in one area may be able to save costs in other areas to still manage a sensible business, see TG2 Read Me 1st [1] for more details.

6.5 Fair Value Reality for Options Trading ("42"?)

The entire question of risk-neutral/arbitrage-free possibilities for options trading requires very much greater technical derivation, as is performed in the chapters below. For the moment one need consider only the basic concepts introduced in Chapter 5. In particular, the essence of options trading is the synthetic replication process. In general, it is not workable to buy and warehouse the entire underlying at the outset. Rather, a certain amount of the underlying is obtained, and then a large number of rebalances are used to fine-tune the position to keep it risk-neutral on a probabilistic basis.

Somehow, one must demonstrate that the synthetic replication process does indeed qualify as the "warehoused underlying risk-neutral" equivalent. As we shall see, this is "provable" but only when a large number of crucial simplifying assumptions hold. Unfortunately, many of those assumptions are either impossible or impractical in the real world. Thus, options traders will need to use some additional methods to account for the inaccuracy in the market convention models.

Since the delivery process synthetically replicates the option by many rebalances in the underlying, liquidity is required through the holding period, not just on the first/last days. Put differently, the risk-neutral process is really only intended for the case where there is liquid underlying instrument "all the time".

As before, this categorises options separately from "pure insurance" or "contingent derivatives" products, which may not have any tradable underlying (e.g. earthquake insurance), and thus require a provisioning approach to hedge the position. It should be clear now that having a probability weighted amount of "cash" on hand will not be so easy to use in the risk-neutral/arbitrage-free process sense, compared to when there is a tradable underlying (i.e. the option and its underlying are naturally offsetting, while an option and a pool of provisioned cash may not be perfectly offsetting).

Moreover, the standard market convention methods only consider the market risk of the underlying price as "risk" (i.e. Delta risk). In reality there are other important market risks (e.g. volatility/Vega risk, etc.).

Even if there was only Delta risk, the market convention methods almost always assume that the dynamic replication process is free of transactions costs. Quite a big assumption for market makers, whose entire revenue depends on their options' bid/offer spread, and from which they must pay for their (many) rebalances.

6.6 Arbitrage-Free Forward Prices (fair value) vs. Actual Forward Prices

With non-contingent derivatives, a Cash-and-Carry-like delivery mechanism can be argued to be arbitrage-free, but only in a "delivery and instantaneous sense". Notably, at the instant in time that a market maker creates a delivery process (e.g. borrow funds, buy and warehouse the underlying, etc.), the funding rate must be arbitrage-free in relation to the market rate of return to the forward date, since that is the only way in which the forward price is arbitrage-free. Mathematically, with continuous compounded funding rates, and no other costs, this arbitrage-free forward price is:

$$F = Pe^{rt} \qquad (6.5)$$

where r is the funding rate, and at this instant in time it must, by arbitrage arguments, equal the market rate of return r_M to the forward date t.

However, this is purely an artefact of the market maker "locking in the delivery" at that instant. In a sense, the market maker does not really care what the forward price after that instant, since the objective of the Cash-and-Carry process is to ensure risk-free delivery (not to consistently predict forward prices). Indeed, the arbitrage-free arguments do NOT apply outside of that instant in time and for that price. Thus, the formula should actually be written as:

$$F_i = P_i e^{r_i t} \qquad (6.6)$$

where i denotes a particular "instant in time".

If a moment later the markets are altered by some supply/demand forces, say, an earthquake, then at the new moment in time new spot prices will be determined by the markets.

Then, one may again apply an arbitrage-free based argument to a new Cash-and-Carry delivery mechanism starting at that new moment in time. Even if the funding rate remains the same, it does not mean that previous funding rate is still arbitrage-free with the current market conditions. That is, the new formula is

$$F_{i+1} = P_{i+1} e^{r_{i+1} t} \qquad (6.7)$$

This (new) formula still produces an arbitrage-free forward price, but now at the instant $i+1$. That is, the new funding rate r_{i+1} must be arbitrage-free in relation to the (new) market rate of return $r_{M(i+1)}$, at the new instant in time.

However, the market rate of return $r_{M(i)}$ (almost surely) may have little to do with the market rate of return at the new instant, $r_{M(i+1)}$. Equivalently, the earlier instant's (arbitrage-free) forward prices F_i may have little do with the new instant's (arbitrage-free) forward price F_{i+1}.

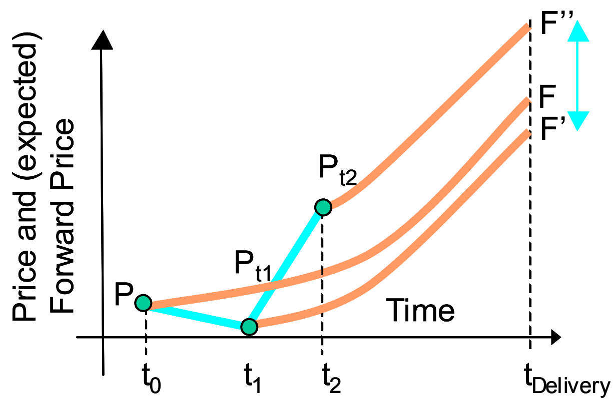

Figure 6.6 – 1. Three different instants of time with three individually instantaneous arbitrage-free spot/forward price combinations, but which are not arbitrage-free over time.

Figure 6.6 – 1 illustrates this point, where at each instant in time t_0, t_1, and t_2 there is instantaneously a spot/forward price combination that is instantaneously arbitrage-free, and thus at that instant the implied market rate of return must equate to the funding rate.

However, the actual market rate of return over this period is defined by the price history of P, P_{t1}, and P_{t2}. Clearly, the actual holding period market rate return as experienced by the price history has little to do with the "instantaneous funding rate implied market rate of return".

Those familiar with bond pricing will recognise this "issue". A bond's price can be converted to its IRR and back. The IRR does not actually tell us anything about the holding period return of the bond. All it tells us is that at that instant, the bond's price implies a particular coupon stream for breakeven. A moment later, the bond's price has moved unpredictably, and so has its IRR, though the new price and new IRR will be "locked" together each instant by Present Value Theory (which is just a fancier version of a Cash-and-Carry argument).