

## 2.1.6 Spot/Forward Arbitrage Example: 3M-Forward Ferraris

The previous Section defined the classic fair value calculation for forwards using the classical arbitrage-pricing concept. This Section will dissect that calculation to illustrate a number of particularly important effects, and especially issues that arise in the real world. The net result will be a number of new rules and considerations for the use (or adjustment) of arbitrage pricing methods.

Suppose that you were approached by a trader who was looking for a forward market in Ferraris<sup>44</sup> since he was expecting a hefty bonus in three months, and wishes to lock in the price today. What forward price would you “show” this client, and how would you manage the process?

Consider two approaches: static replication/delivery, and dynamic replication/delivery.

Static Replication/Delivery: this process follows exactly from the derivations in Section 2.1.5.4 above. If a (spot) Ferrari costs GBP 110k, then you will need to borrow that amount at the current rates (say 3-month LIBOR<sup>45</sup> +  $x$ ). There will be additional costs (which also need to be funded) to cover warehousing, security, etc. Finally, don't forget that you must also be paid for providing this forward Ferrari service, so your fees/profits must also be included in the “holding period costs”. Making some assumptions about these costs, leads to:

$$\begin{aligned}K &= P(1 + rT) \\K &= 110,000 \left( 1 + \left( \text{LIBOR} + x + \text{Warehouse} + \text{Security} + \text{Fee} \right) * \frac{90}{360} \right) \\K &= 110,000 \left( 1 + \left( 0.0484 + 0.0100 + 0.0050 + 0.0020 + 0.0010 \right) * \frac{90}{360} \right) \\K &= 111,826\end{aligned} \tag{2.2}$$

Therefore, according to the arbitrage based fair value arguments, this is the “fair” forward price of the Ferrari for 3-month delivery.

However, is this really so? Consider just two of the important factors: default/credit risk, and impact from 2-way flows due to competition.

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<sup>44</sup> You may wish to substitute another deliverable such as a villa in Spain, a barrel of oil, some foreign exchange, a bond, an option, or whatever.

<sup>45</sup> If you don't know what LIBOR is, don't worry it's introduced in Chapter 7 and then in much greater detail in TG2 Bonds & Swaps. For now just consider it to be a key lending rate.

- a) Default Credit Risk: if all goes well, at the end of the three months you show up with the Ferrari, and your client shows up with the cash and the deal is settled. However, suppose that your client fails to show. You are now stuck with a Ferrari and no way to pay back your loan. If there is a liquid secondary market in Ferraris, then you could sell into that, and take whatever price you can get (in some cases you could actually make a lucky profit this way).

Clearly, the arbitrage pricing methodology above did not account for this type of risk. Nevertheless, it may still be possible to adjust the forward pricing calculation to account for it. The most common approach employs a statistical equivalent cost that is based on the probability of the default, and the likely size of the loss that is incurred during a default, and then charging a probability weighted equivalent amount. For example, if all of the default risk could be expressed as (on average) 1 default in every 100 forward deals, and that each default (on average) costs GBP 10k, then the forward price above could be adjusted by  $10k * 1/99$ <sup>46</sup>.

- b) Competition and 2-way flows: remember that arbitrage pricing is only meaningful if there are liquid two-way flows. So arbitrage based prices may not hold in the real world (e.g. due to the costs different from yours incurred by the other traders). Suppose that a large bank also made markets in 3-month forward Ferraris. It is likely that their borrowing costs would be much smaller than yours (e.g. they might be borrowing at LIBOR flat, compared to “your” LIBOR + .0100 assumed here). This means that the forward price they would calculate would be 0.0100% (also referred to a 100 basis points or bps) lower than yours would be. Is this arbitragible? No! Its not arbitragible since in order to “arb” the difference between your forward price and their forward price, you would need to be able to sell at your forward price and simultaneously buy at their forward price. However, the cost of your delivery is the higher price so you end up loosing the difference (probably 100 bps of funding, since for you to generate a short delivery you would be lending the funds and would likely receive only LIBOR, or worse).

This is a very important consideration since there will be many occasions wherein there will appear to be an arb “on the screens” but as soon as you try to transact “both ways” you will find that the bid and offer prices are sufficiently far apart.

Dynamic Replication/Delivery: thus far all of the arbitrage-based calculations have been “static” in the sense that all costs and prices are locked-in in advance for the

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<sup>46</sup> Its 1/99 rather than 1/100 if you do not expect to collect the “default risk fee”.

entire delivery process. However, in the calculation above the single biggest cost was 3-month LIBOR at 4.84%. Suppose that the overnight (O/N) borrowing rate is 4.22% and thus 62 bps less than the 3-month rate. In this case, it would be possible to borrow the GBP 110k for the spot Ferrari at the O/N rate, and then keep rolling the O/N loan every night for the 3-month duration, thereby saving 62 bps (put differently if all else remains unchanged, then your fees/profit goes from 10 bps to 72 bps).

Sound good? Is the massive increase in profits an attractive proposition? Well, yes and no. This approach is no longer an arbitrage free pricing mechanism since the position is now exposed to market risk. Namely, there is no guarantee that the O/N borrowing rate will remain constant over the entire period. Thus, the dynamic process is in part a directional play on the borrowing rate (and so clearly implies a forecast and market risk, whether the trader knows it or not).

So should dynamic replication be disallowed? Definitely NOT. There is nothing wrong with taking market risk so long as:

- 1) Your mandate permits taking this type of risk.
- 2) You and the shareholders are being fairly compensated for taking this extra risk (more on this later, and see Section 2.1.8 and Chapter 3.2.6 for an introduction to risk adjusted returns)
- 3) And related to 2), you alter your costs/prices and provisioning accordingly, since the basic arbitrage calculation no longer applies to you or your regulators etc.

ASIDE: As a point of interest, most investment banks/banks prefer to centralise their funding for various operation reasons. This means that the trading desks do not manage their own funding. Invariably, the funding desk provides them with O/N funding (in spite of the fact that 99.9% of the deals in a typical trading book are much longer dated). This means that the dynamic vs. static issue used above is quite real. Indeed, matters can be rather more interesting since not only is the funding provided on an O/N basis, but often the accounting process is easier if O/N rate is assigned on a monthly or some average basis. Clearly, this type of accounting will further complicate the arbitrage pricing considerations.

Please be sure to take extra time to digest this Section. It may seem all too simple now, but please pay extra attention to the issues discussed here. This is especially important for the liquidity and static vs. dynamic items. These can introduce P&L difficulties in subtle and mischievous ways later on when dealing with products that are more complex and particularly those relying on dynamic replication for their valuation (e.g. options).