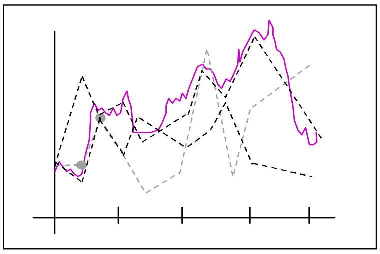
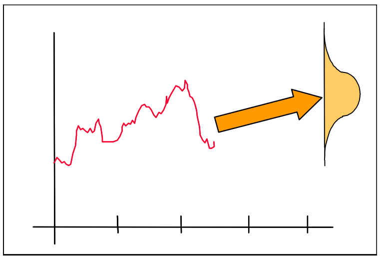
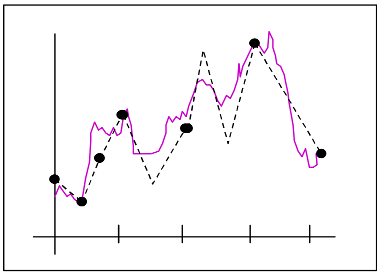
8.7 Summary: A First "Complete" Model for Valuation Under Uncertainty







The story so far: the objective was to forecast prices for the purpose of valuation and risk management. A statistical approach was chosen for the practical reason that pure prediction of prices is considered intractable. A two-component forecasting machine was proposed: one component forecasts the drift, and one component forecasts the uncertainty. The development of these components saw many choices in the process of developing a first model, and each of the turns taken when such choices were made were accompanied by discussions on the implications and references were provided for more detailed analysis in later Chapters and other TG2 books.

It was also decided, that although many different possible forms of the model may be developed, for now only two forms are considered: an arithmetic (Normally distributed linear growth prices) form, and a returns or geometric (Log-Normally distributed exponential growth prices) form. Initially these models were presented on a "pseudo" forecasting basis to assist with an intuitive development. However, in creating a practical forecasting machine, these forms were converted into differential and difference forms. The differential form of the returns (geometric) process is considered the most common market convention model, and is:

$$dP = \underbrace{P(r-q)dt}_{drift} + \underbrace{P\sigma_r dz}_{uncertainty} \qquad (8.37)$$

or

$$\Delta P \cong P(r-q)\Delta t + P\sigma_r \Delta z \qquad (8.38)$$

where the drift term shows all of the effects of the returns requirements r, the income from any possible income stream (coupons, dividends, etc), and the adjustment due to Ito's Lemma arise later during the solution step.

The arithmetic or absolute (price) based forecasting model equations are:

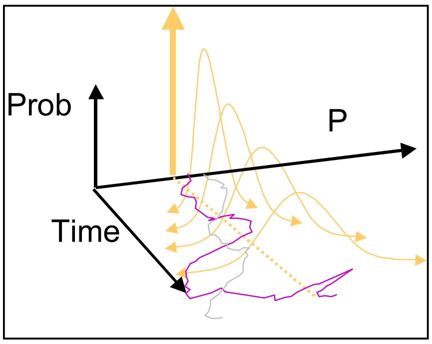
$$dP = P(r-q)dt + \sigma_P dz \quad (8.39)$$

or

$$\Delta P \cong P(r-q)\Delta t + \sigma_P \Delta z \quad (8.40)$$

In these expressions, the variable z contains one possible choice for the model of uncertainty. In particular, the forward price variability is random with either a Log-Normal distribution (for the geometric process) or a Normal distribution (for the arithmetic) on any particular forward date, and that the width of the this distribution stretches according to its standard deviation and by $\sqrt{t}$.

The nature of the drift, the shape of the distribution, and the evolution of uncertainty have all been chosen to reflect real world attributes, though to some extent the choice of these first models has also been influenced to ease the mathematics. Nevertheless, they are a reasonable and practical first choice, even though the true test of their "goodness" can only follow from P&L verification as applicable to a specific business mandate criteria.



As noted the common decision is to use the geometric form since the resulting Log-Normal forward prices cannot be negative. Though again, a P&L verification should be used to make this decision as in some cases the error introduced by allowing negative forward prices by a Normal distribution may be less than then the error of using a Log-Normal but positive forward price model (e.g. see Chapter 12).

The models are "calibrated" to the real world in two ways. First, the shape of the distribution is chosen to reflect (sufficiently closely) a shape that is seen (or expected to be seen) in the real world (and based on traders' intuition and mathematical insight). Second, the parameters of the chosen model can be calculated from real world data (e.g. "historical volatility" calculations) to calibrate the chosen model's parameters to the market.

The solution or actual pricing/risk management calculations require the integration (summation) of the differential (difference) equations above. Loosely speaking, one may think of the integration in two parts. One of the integrations (summation) is over time and provides a forecast of the average forward price (or average pay-out). The other integration is over uncertainty (distribution) and accounts for the impact of variability in the forecast.

An analytical solution is preferred, but this is not always possible or practical, and a numerical method may be required. It is important to understand the limitations of the numerical methods as approximation errors may arise in unexpected ways.

The solution provides a summation or averaging of the expected payout, but it does so on the forward date. The complete solution usually also requires that the integration results be present valued.

The differential/difference forms of these models correspond to generate single steps in along a path of forward prices, which will almost surely not produce the true forward prices. Suitable “solution” techniques, and assuming that the models have been verified to be market and P&L consistent, should produce forward prices that are “on-average” correct, and forward price variability that is on-average consistent with the market’s variability. Hopefully, all of which results in “on-average” P&L’s consistent with the models assumed rate of returns (drift) and volatility (uncertainty). However, to achieve this the trading must also be performed in a “consistent” manner, since a statistical valuation model is meaningless to the trader/investor doing only one or a few trades.

This type of valuation model does not really care what the traded instrument is. All the solution provides is a probability weighted average of the pay-out function.

Importantly, however, the models thus far include a risk premium parameter. This parameter is measurable, but is likely to be different for each and every trader/investor since it reflects subjective risk preferences. This means that valuation results will be meaningful only to those with the same risk preference (though it has been promised that there is a kind of “fix” for this in later Chapters).

Finally, a complete development process should include model verification and even “model audit” so the P&L performance of the model may be tested against actual or expected trading and market scenarios to ensure the best model for the trading mandate (e.g. see Chapter 12). Obviously if these models fail your particular P&L tests, as required by your particular business mandate, then another model must be developed if possible, or at least a model that incorporates sufficient additional features to account for greater accuracy or market or securities/derivatives affects (e.g. price gaps, mean-reversion, etc)