9.1.6 Basic Sensitivity Hedge: Options Delta & Delta/Gamma/Rho (Pyramid) Hedging

This Section provides introductory calculations and considerations for hedging a simple options position and specifically concentrating on introductory issues dealing with sensitivity rebalance ratios. The important notions here are the use of "multiple" sensitivities in highly non-linear positions, rather than any attempt to provide detailed options trading details.

Thorough treatment of real world options position keeping methods and issues is provided in [6], [8], [12], and other books in this Series. However, details of trading strategies and hedging techniques seen here are also discussed in Chapter 3.2.5, and in Chapter 10.2 in more detail, where shortcomings of valuations models and assumptions about the connection between rebalance rations and holding period P&L are highlighted.

Three cases are considered here:

- First, there is a simple V01-like (Delta) hedging case for a simple option position. This example will show that options may be so non-linear that in many cases the higher order risks (curvature) must also be hedged, and so higher order sensitivities are used to calculate additional hedge rebalances. Please keep in mind that there are many aspects and interactions in options trading that are crucial, and which are deferred to the relevant TG2 books in this Series.

- Second, there is a simple (Gamma) hedging rebalance example, which shows one important method to alter the position curvature risk.

- A third case is used to provide a first glimpse at the larger problem of multi-dimensional interacting risk hedging issues. A technique called "Pyramid Hedging" is introduced as one possible strategy for dealing with multi-dimensional coupled risks.

9.1.6.1 Basic Sensitivity Hedge: Options Delta – Long Position

Begin with the 110-strike call option position from the earlier Sections, and assume that you own 20 contracts, each with a notional underlying value of 1.00 for simplicity. The underlying market is currently at 100, your funding cost is 4% (continuously compounded annual - CCA[326]), the dividend income 6 % CCA, the market volatility is 18%, and the options expire in 1-year. Assume also that Equation (10.13), from Chapter 10, is a credible valuation formula for these options.

Assume that this is an option on a "linear" underlying instrument. This includes a wide range of assets/contracts such as equity indices (e.g. the S&P), FX, almost any physical commodity (gold, Frozen Concentrated Orange Juice (FCOJ), etc), etc.

Your boss tells you that you must keep this options position "Delta neutral". The first step is to calculate the position Delta. You could use a direct analytical calculation for this, but for the moment the Delta is approximated from a V01-slope based calculation, thus the "per contract" approximate Delta is:

$$V01-slope\ Sensitivity\,(up-shift) = \left.\frac{\Delta V}{\Delta S}\right|_{Up-Shift} = \frac{V_{u+1}-V_u}{\Delta S}$$

$$= \frac{V_{100.10}-V_{100.00}}{0.10}$$

$$= \frac{BS(100.10,110,4\%,6\%,18\%,1)-BS(100.00,110,4\%,6\%,18\%,1)}{0.10}$$

$$= \frac{2.855075\ -\ 2.827584}{0.10}$$

$$= 0.27491$$

However, there are 20 contracts, so the total position Delta is 20*0.2749 = 5.486.

The rebalance that is required is one that provides a Delta of approximately -5.5 to cancel out the current approximately +5.5. Options theory is covered in detail in [6], but what is important for this example is that the option's Delta is also equal to the number of underlying contracts that will make the position Delta neutral. Therefore, the rebalance that is required here is to sell 5.486 of the underlying contracts.

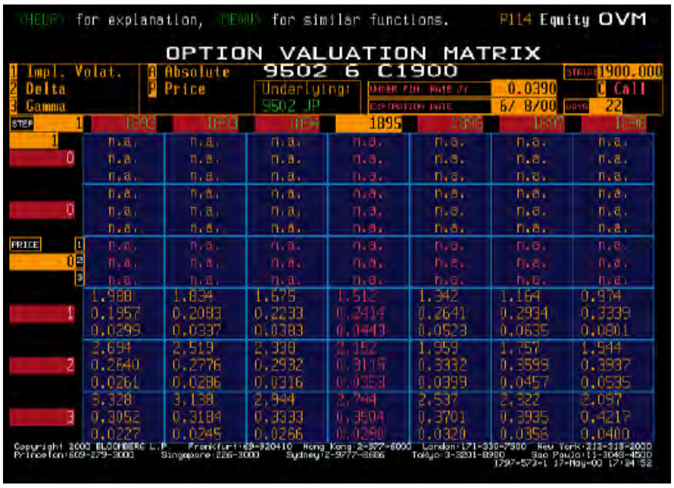
Of course, this is not possible since contracts must be sold in "modulo" or "integer" amounts[327]. This means that now there is a decision to be made. Do you wish to be over hedged (by selling 6 contracts), or under hedged (by selling 5 contracts)? Usually, this

[326] CCA rates hardly ever arise in practice as integral to contract quotation, but they are very common in derivatives formulations such as many options models. Here CCA is used for simplicity, and remember, as pet Chapter 7, that conversion from CCA to any other quotation (frequency) basis is quite easy.

[327] Many OTC transactions can be negotiated to be in exactly any amount that the client wishes. Thus if the underlying instrument in this example was an OTC trade, then it would be possible to negotiate for a notional amount that is equivalent to 5.486 contracts. However, this is generally not done in practice. Rather, there are "unofficial minimum size" trades. For example, a particular swaps market you may not wish to deal in anything less than a notional of 5 Million, or at least charge extra for the exceptions.

decision is subjective and will depend on the trader's view of the markets, transactions costs, etc. (another example of skills and worries that comprise a trader's job). Suppose that the initial decision is to sell only 5 contracts, because on the balance of probability you expect a greater likelihood of a "rally" than a "dip". This means that after the rebalance, the position's (approximate) Delta is not actually flat, but rather has a residual risk synthetically equal to being long 0.486 of the underlying contract[328].

Here is an example options calculation from one of the Bloomberg analyses screens.



So how would the P&L perform if the market fell 5 points?

The position report on inception date is:

Contract	Term	Strike	r	d	v	Market	Contract Price	NumCntrcts	Position Value
Option	1	110.00	4.00%	6.00%	18.00%	100.00	2.8276	20.00	56.55
Underlying						100.00	100.00	-5.00	-500.00

The position report after a 5 point drop in the markets is:

Contract	Term	Strike	r	d	v	Market	Contract Price	NumCntrcts	Position Value
Option	1	110.00	4.00%	6.00%	18.00%	95.00	1.6728	20.00	33.46
Underlying						95.00	95.00	-5.00	-475.00

So, the P&L impact is:

[328] This is approximately 10% of the target position size and so may be too large a trading mismatch for some mandates. In real world trading, and especially in larger operations, where much bigger size is traded, the residual risk is a much smaller relative to the position size. However, in smaller trading operations, or in certain contracts/assets there can be this type and size of residual risk, and so may be a significant impact on the business mandate.

Original	Shifted	Difference
56.55	33.46	-23.10
-500.00	-475.00	25.00
		1.90

Again, the "hedged position" value changed, and dramatically so. Notice that relative to the original value of the option position (56.55), the total P&L has changed a cash amount of 1.90 (or 3.4%). This implies that the hedge efficiency is not very good. Again that "we made money" is to some extent "misleading", since if the hedge had been effective, then there should have been no change at all in the P&L.

In fact, had the market dropped further, say by 20 points, the change in the P&L would have been +47.17, which, relative to the original value of the option position (56.55), is an 83.41% return. This implies that the hedge efficiency is very bad, and you should be really "scared" by now since how could a hedged position make so much money. The first thing you should ask yourself is "are there conditions wherein this position could lose this or greater levels of funds?"

What is even more disconcerting is that if we had tested this Delta efficiency for the case of an upswing in the markets, then again the hedged position would make money. No matter what market prices do, you (appear) to make money. The bigger the movement in the market, regardless of direction, the bigger the P&L benefit. How can this be?

This is the magic of options. Figure 9.1 – 9 below illustrates why this hedge is "not working" (or at least why it appears to be a "money machine"). The graph illustrates the individual values of both the options position (curved blue line) and short underlying hedge position (straight red line having a slope exactly negative of that of the options Delta).

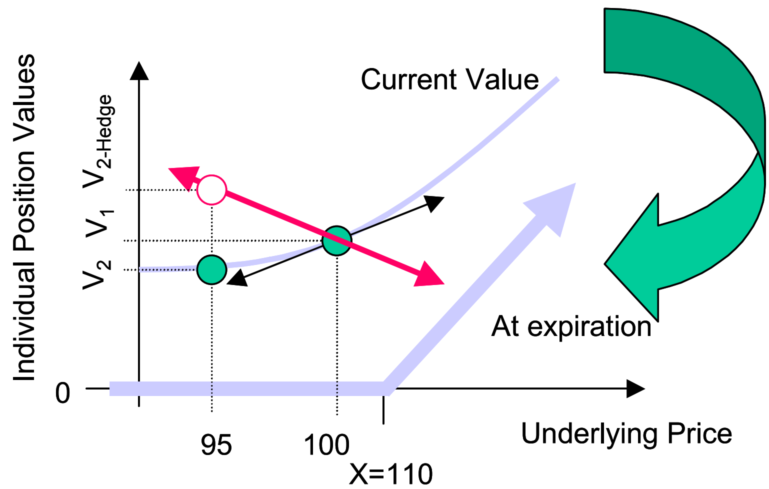


Figure 9.1 – 9. The pay-out profile for the target option position and showing the Delta rebalancing requirements.