11 The Answer is in the "Root"

Root finding, or "solving for zeros" is an important numerical technique used when it is difficult or impossible to obtain an "inverse" answer by other means. Important examples of the application of root finding in trading and risk management include:

- IRR/Yield from bond Price.
- Implied volatility from options price.
- Intermediary step in many American and Exotic options valuations.
- Important component of certain term-structure models.

The technique also makes an appearance in various forms of "optimal" risk/return portfolio allocation/immunisation methods.

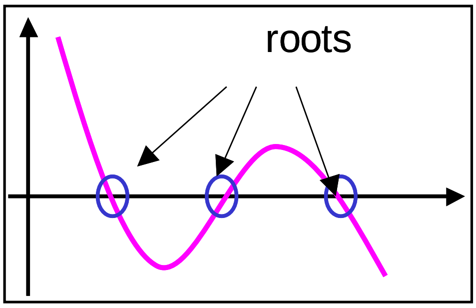
Hidden from most trader's and risk manager's view is the application of these methods in many advanced parameter estimation techniques.

Normally, functions are provided with inputs to produce an answer. However, in many situations, the "answer is given", and the formula must be used in an inverse sense to discover what "inputs" would have produced that "given answer".

When the formula is non-linear, the inverse problem may not be amenable to algebraic or analytical solution, and then numerical methods are required. In some cases, even if there may be the possibility of an analytical solution, the numerical solution may be more "cost effective".

11.1 The Basic Idea

The "root(s)" of a function is the location(s) at which function crosses the abscissas (also called the x-axis or the "0"-axis). The image to the right illustrates a highly nonlinear function that (over the range shown) has three "roots".



There is no guarantee that a general curve will cross the x-axis.

Some non-linear curves may be inverted analytically. For example, the present value of a single future cash flow (e.g. a zero coupon bond), assuming discrete compounding (see [1] or [4]), is:

$$P = \frac{C}{(1+r)^n} \tag{11.1}$$

where C is the amount of cash to be paid on the future date, r is the interest rate or yield, and n is the number of compounding periods.

In this case, knowing the price of the loan would permit an analytic solution for yield since the equation can be inverted algebraically as:

$$r = \sqrt[n]{\frac{C}{P}} - 1 \tag{11.2}$$

However, a traditional bond structure, such as

$$P = \sum_{i=1}^{N} \frac{C_i}{(1+r)^i} \tag{11.3}$$

generally cannot be inverted algebraically, and so numerical methods are used[55].

The numerical root finding methods rely on clever "guessing" strategies that test the "target" function's proximity to zero.

For example, given a bond price P and the bond pricing formula in Equation (11.3), the following construction is possible:

$$\begin{aligned} & P = \sum_{i=1}^{N} \frac{C_i}{(1+r)^i} \\ & \Rightarrow P - \sum_{i=1}^{N} \frac{C_i}{(1+r)^i} = 0 \\ & \Rightarrow f(r) = 0 \end{aligned} \tag{11.4}$$

[55] A limited number of variations, such as those with quadratic form, do admit closed form solutions. Remember the old quadratic roots formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for the roots of $y = a + bx + cx^2$, and there are a few more, but quite limited.