12.6.4 A First Formulation of the Evolution of Uncertainty

Based on the intuition and technical considerations from above, the choice for a formulation of the evolution of uncertainty is one that essentially "stretches" a single "base" shape or distribution by an amount that is proportional to the elapsed time to the forward dates of interest.

For example, if the distribution of choice is the Standard Normal distribution *N[0,1]* (meaning that it is a Normal distribution centred on "0", and having a standard deviation of "1"), such as in the Figure 12.6 – 2 a), then at a later time *t* the forward distribution would be *N[0,f(t)]*, where *f(t)* is a formula for standard deviation that depends on time *t*. For convenience, *f(t)* has been assumed to be $\sqrt{t}$ in Figure 12.6 – 2.

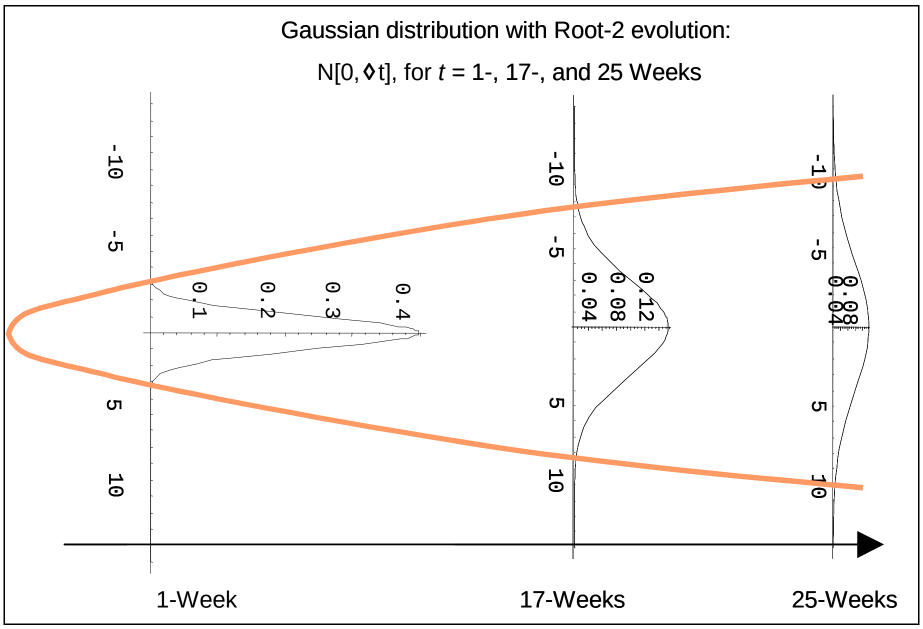

Figure 12.6 – 2 a). Three segments along a continuum of Normal Distribution *N[0,f(t)]*, using a time dependent standard deviation *f(t)*. This is equivalent to a Standard Normal distribution (*N[0,1]*) stretched to have its standard deviation at any point in time becomes **s** = *f(t)*. Notice that the "stretched" distribution is also "shorter" to preserve the area under the curve (as 1).

Or, in 3-dimensional view as:

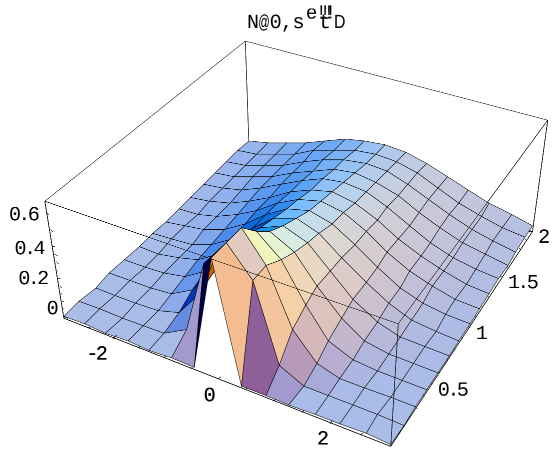

Figure 12.6 – 2 b). The Normal Distribution *N[0,f(t)]*, in 3-D view when $f(t) = \sigma\sqrt{t}$, with $\sigma = 1$.

A general approach for the formulation of the "time stretching factor" is to define variance or standard deviation proportional to time, with a proportionality constant reflecting the process as:

$$Std.Dev = \sigma(t) = at^b \tag{12.17}$$

If the power *b* is ½ and the proportionality constant *a* is 1, then this expression collapses to $\sqrt{t}$ and hence the Root-2 proportionality from above. Obviously there are other possible choices for *a* and *b*, and using other choices may improve the model performance (in terms of P&L). These choices generally lead to more complex abstractions and mathematics, and may require more than novice trading skills/expertise (see for example Chapter 17, and in [2] and at [14])

So accepting the technical and intuitive arguments in favour of using the Normal distribution in combination with a Root-2 time evolution process, the uncertain component of the forecasting expression becomes:

$$P_{Forward} \triangleq P_{Today} * e^{\tilde{r}t} + N\left[0, \sigma_P \sqrt{t}\right] \tag{12.18}$$

or

$$P_{Forward} \triangleq P_{Today} * e^{\tilde{r}t} + N\left[0, P_{Today}\sigma_r \sqrt{t}\right] \tag{12.19}$$