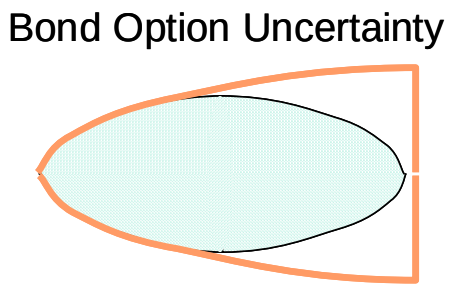
14.4.1.4 Bond Options

Another example of a model uncertainty complication due to contract specifications is that of the "pull-to-par" effect seen with bond options. Consider that the evolution of uncertainty in the standard model is of a perpetual increasing width due to the Root2 effect built into the Brownian motion. Though it is true that there will be uncertainty about a bond's price, and that this uncertainty will increase, it will eventually need to "decrease".

Consider that at maturity, the bond's price is known with certainty (it's the Par value printed on the bond itself). Therefore, as one approaches maturity, the level of forward uncertainty is actually decreasing, since the bonds value is being "pulled to par".

Thus, the bond price uncertainty increases initially, but decreases once one is sufficiently close to maturity, resulting with an uncertainty envelope shown by the shaded region in the image to the right. The orange line shows the uncertainty envelope (+/- 1 SD assumed by the standard model.



Clearly, after some sufficiently long period, the discrepancy between the standard model and the actual bond uncertainty will be too large for any P&L. For example, a 9.5-year option into a 10-year bond will be given a very high option value by the standard model, when in fact it would be virtually valueless.

This is not a feature that can be "washed away" with term-structure adjustments (or with an common, but erroneous method used in capital market to base the valuation on the forward rate of "floating side" of swaptions).

However, it is possible to derive "correct" mathematics to deal with matter, and with a reasonably cost manageable process. One strategy is to use a "two-pass" valuation that evolves interest rates, and then (with the second pass) incorporates the "price based" valuation aspects of the problem (see [8.b] for details) as shown in Figure 14.4 – 1.

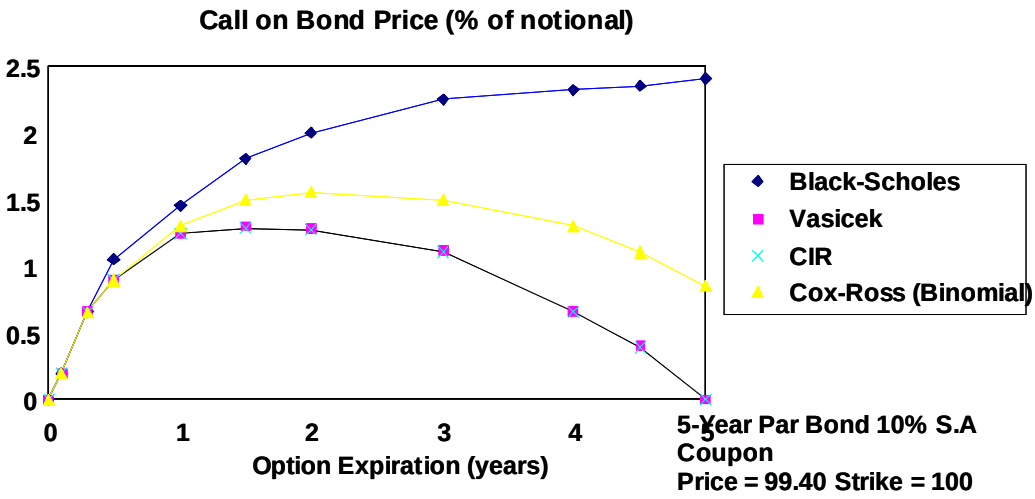


Figure 14.4 – 1. Model with varying success of representing bond option pull-to-par (though much of the pull-to-par "success" is due to the specific manner of the implementation, not just the choice of model).

Here it can be seen that there are implementations that can (apparently) correctly produce the pull-to-par effect with "proper" valuation methodology, and relying on the Gaussian/Root2 machinery[159].

However, notice that the difference between the "standard" model and the "fancy" models is quite small for options that have expirations less than about 20% of the life of the bond. This means that for those trading short dated options (that is most of us), the standard machinery maybe "good enough". Of course, this assessment should be based on a P&L analysis for your circumstances and mandate. Still, if it the case, then there may be no need to implement a big/complex valuation technology.

[159] It cannot be emphasised too strongly that the "goodness" of the results, especially for the Vasicek and CIR model curves, is more to do with the manner in which they are implemented, rather then the "core" model specifics. That is, although Vasicek and CIR have important models, here the pull-to-par facility is more to do with the "2-pass" valuation procedure than the model themselves.