

## 15 Introduction to Time Series Analysis and Dynamics

This Chapter introduces methods for Time Series Analysis (TSA). These methods attempt to model the entire dynamics of a process, and often aim to capture the “directional” components of market behaviour. The directional components can be in terms of outright prices and returns, or for secondary measures such as volatility, term-structure curvature, or anything that is tradable or of interest.

A detailed treatment of TSA is provided in [3.f].

TSA “looks like” many of the curve fitting and characterisation methods discussed in Chapters 5 - 8. However, in those chapters the idea was to interpolate a data point “inside” the range of dataset, or to extract “shape” of the data inside the range of the dataset, here similar machinery is used to extrapolate outside of the range of the data, and with the important distinction of predicting into the future.

The study of TSA, roughly speaking, falls into two categories: Econometrics-like methods, and dynamics-like methods.

- Econometrics-like methods use statistical and Maximum Likelihood Estimators (MLE) technology to model TS data. Much of the machinery used here overlaps with the methods introduced in Chapter 8 and in Chapter 17.

These methods can be further classified as “proper” Econometrics or “fundamental<sup>168</sup>” models. Fundamental based TSA begins with a “model” derived based on some “fundamental economic” arguments, and then “fits” the parameters of that model. Pure econometrics, on the other hand, is more of statistical analysis without an a priori model.

- Dynamics-like methods aim to identify the “driving forces” buried in the TS data. This can be either with an a priori model and then apply parameter estimation, or with something resembling a pure “curve fitting” procedure. The technology does not rely on statistical characterisation.

Most of the methods in this category are geared toward identifying periodic or near periodic behaviour. The methodologies can be further classified in this sense. The classic (Fast) Fourier Transform (FFT) related methods are suitable for TS data with

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<sup>168</sup> Here, the word fundamental is used in the econometrics sense, rather than the trading “fundamental analysis” sense, though the two meanings are related.

periodic behaviour. By comparison, the methods from Non-Linear Dynamics (NLD), such as fractal and chaotic methods, are suitable for “aperiodic” data (i.e. data that appears to have periodicity, but “not quite”).

This Chapter examines each class of these methods, and provides introductory examples. There is a great deal of analysis that can be performed with such methods, and each of these methods may “appear” to “work” (i.e. predict the markets) under “some” conditions. However, the key is to have a methodology that is not only reliable and consistently correct, but also is profitable (i.e. forecasting and profitability are not always the same thing). There simply is no substitute for real trading experience, though PaR based simulation can be very helpful.

The testing of the reliability of these methods should include both in-sample and out-of-sample tests. For example, once a forecasting equation is produced, it can be tested to see how it performs with the data that was used to create it (i.e. in-sample tests). Additionally, it is helpful, when possible, to keep aside some of the data (i.e. not used in the parameter estimation step), and then compare the forecasting performance against that as well. This corresponds to traditional “back testing”.

## 15.1 Time Series Analyses – Econometrics

Strictly speaking, econometrics need not be restricted to TSA, as it can be used to examine so-called “cross-sectional” problems. Here, however, the focus is on its application to TS data.

In crude terms, econometrics is (usually linear) regression (such as Least Squares seen in Chapter 10) used to fit a model equation, which is then used for forecasting or prediction. Two types of econometrics are considered here:

- “Fundamental” econometrics begins with a “proper” fundamental model (e.g. expressing the relationship in, say, macro economic factors such as GDP, inflation, etc) derived from basic principals thought to describe the process. For example, the Capital Asset Pricing Model (CAPM) may be considered a fundamental econometric model.
- “Pure” or empirical econometrics is simply an analysis to produce a forecasting equation that does not presume an explicit fundamental relationship. For example, the Arbitrage Pricing Theory (APT) model may be considered a pure TSA. Here, the model is exploiting “technical characteristics” of the equations.

The same econometric/regression machinery is used for both, though the analysis and (iterative) improvement in the two approaches have different requirements.

### 15.1.1 Traditional Time Series Analysis – Simple

A simple example is instructive. Suppose that you had a history of, say, prices for a traded instrument, and wished to produce a forecasting machine. A very simple “guess” for a model or forecasting machine might take the form:

$$y_t = a + by_{t-1} + \varepsilon_{t-1} \tag{15.1}$$

where the  $y$ 's are “today's” and “tomorrow's” (or the next period's) prices, and  $a$  and  $b$  are parameters to be estimated from the data. The  $\varepsilon$  is an error or residual term, since almost surely, the forecast will not be exact.

A regression or similar technology can be used to “fit the parameters”. One way to imagine that procedure is to consider that the technology can repeatedly compare each price pair in the history (assuming they are  $t$  and  $t-1$  apart) and keep track of the  $a$  and  $b$  that makes  $y_{i+1} = a + by_i$ . Then, choosing the best  $a$  and  $b$  combination that (on average) produces the “best” results (in a statistical sense).

For example, Figure 15.1 – 1 shows the 1<sup>st</sup> Return map for a S&P history. This plot corresponds to a “prediction” of  $y_i$  by  $y_{i-1}$ , and so a linear regression on this plot would produce the desired coefficients for Equation (15.1). An ANOVA analysis of that regression would provide insight into the “goodness of fit” and related assessments.

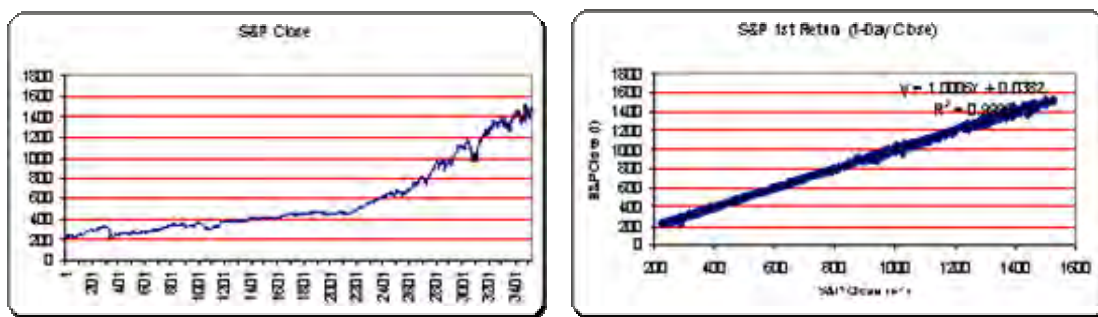


Figure 15.1 – 1. 1<sup>st</sup> Return Map for 10-years of S&P Closing Index values.

The regression fitting procedure produces a formula as:

$$y_i = 1.0005 y_{i-1} + 0.0382 \tag{15.2}$$

with  $R^2 = 0.9995$