15.2 Time Series Analyses – Spectral & (Non-Linear) Dynamical Analysis

Almost all of the methods considered thus far use a statistical framework for modelling and characterising data and processes. This Chapter introduces TSA from a spectral analysis and (non-linear) dynamics perspective. These TSA methods are in sharp contrast to the statistical TSA methods of Section 15.1 in that they aim to model the “underlying” or “inner” specific driving processes, rather than modelling the “outside” or “statistical characteristic” of the process.

Almost exclusively these methods focus on processes or data properties that have exact or “near” periodic features. Purely random processes are not amenable to the methods in this Section. However, and this is important, these methods can determine whether a process is random or merely “looks random”.

Thus, much of the discussion here relate to methods that characterise process as determinisitc, or at least only partially random. The non-random components are then subjected to further analysis to decide if they are exactly periodic or “aperiodic”.

Exactly periodic methods may be analysed and modelled with classical spectral analysis, such as (Fast) Fourier Transforms (FFT’s).

Aperiodic process may be analysis and modelled (to some extent) with methods from Non-Linear Dynamics (NLD). It is important to emphasise that even when NLD methods are exactly applicable, the results can be extremely sensitive to data accuracy, and so a “perfect” model may produce unreliable forecasts simply due to extreme dependence on round-off errors or other approximations. Nevertheless, they do offer the very attractive possibility of identifying and modelling processes that appear to be random, but have (at least some) structure “buried” in the data.

As such, the key issues and derivations are:

- Identifying structure in Random ”looking” data, or equivalently, when is random “looking” data not actually random.
- How to identify periodic vs. aperiodic datasets.
- Modelling (exactly) periodic process (e.g. FFT’s)
- Modelling aperiodic process (e.g. fractals, chaos, etc)
- Predictability vs. accuracy becomes and issue due to extreme sensitivity of the models to input data accuracy.

After all that, there is the practical question of how and where these results may be used in trading and risk management. As with Chapters 12 - 14, there is the possibility of applying these results to any of all of:

- Creation of outright forecasting models (e.g. drift + uncertainty + NLD adjustment).
- Derivation of "risk neutral" and arbitrage free machinery for the valuation and position keeping of securities and derivatives under "quasi" uncertainty.
- Simulation of positions or portfolios with full market modelling and trading strategies for the assessment of risk-adjusted holding period P&L (i.e. PaR).

15.2.1 When is a price history "truly" random, and when is it predictable?

Figure 15.2 - 1 a) – d) shows four "price histories". What can be said about the predictability of such time series purely by inspection? Can you tell if any of them are:

a) Real (market data) vs. formula/synthetically generated?
b) Purely Random?
c) Exactly Predictable?

The answers are provided further below.

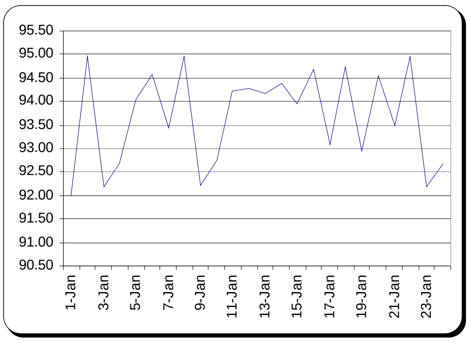

Figure 15.2 - 1 a)

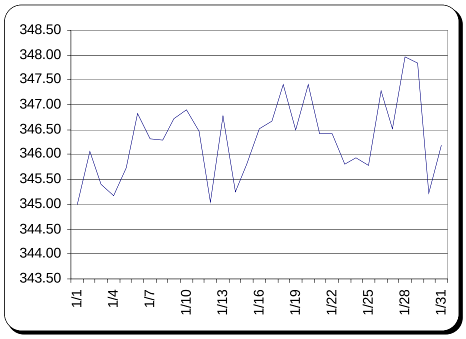

Figure 15.2 - 1 b)

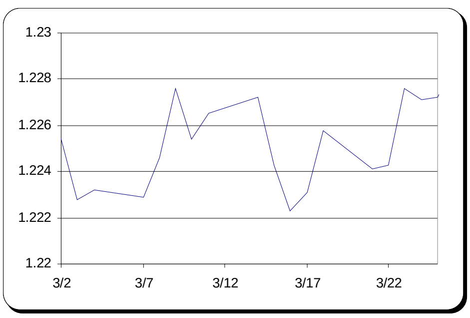

Figure 15.2 - 1 c)

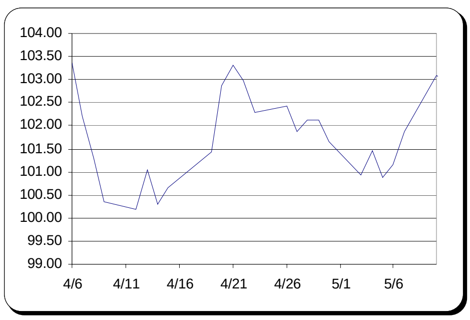

Figure 15.2 - 1 d)

15.2.2 Real vs. Synthetic and Random vs. Predictable Data

For now begin with the notion that there are only two possibilities: a purely random price process, and an exactly predictable price process[173]. Then, consider the question: is it possible to have an exactly predictable price process that "looks" like a random process? If the answer is yes, then check all random looking processes to see if they are predictable.

Figure 15.2 - 1 a) and b) illustrates two price histories which on "first inspection" look as if they could have been extracted from actual price data. In fact, they are both "synthetically generated" price histories. Figure 15.2 - 1 c) and d) are real price histories (CAD/USD and S&P500). However, is "just eyeballing" these charts is not sufficient to reveal if they are real, or indeed if they are predictable or random?

It may be surprising to discover that Figure 15.2 - 1 a) is "exactly predictable", while Figure 15.2 - 1 b) is "purely random"[174]. Thus, if Figure 15.2 - 1 a) represented a real market, then it would be possible to predict with (near) certainty "the forward price". If, however, the market followed Figure 15.2 - 1 b), then predicting tomorrow's price would be no easier than predicting the next role of the dice.

The series shown in Figure 15.2 - 1 a) was generated by a well known (deterministic) equation called the Logistic Equation[175]:

$$P_{i+1} = 4\lambda P_i\left(1 - P_i\right) \tag{15.15}$$

where λ is a constant, and P_i is the "current price" while P_{i+1} is a forward price.

This is a very famous equation in the study of non-linear dynamics, for amongst other things, it can be used to demonstrate that a very simple deterministic expression can result with what otherwise appears to be a random or stochastic process, when in fact it is predictable. The Logistic equation will produce "random looking" series for specific choices of λ. For example the series shown in Figure 15.2 - 1 a) was produced with λ = 0.9846. Notably, for values of λ less than approx 0.83 the series will not appear random at all. The *whys and wherefores* of this (interesting) behaviour are not too complicated, but are deferred to [3.f].

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[173] Equivalently, these could be referred to as purely stochastic, and purely deterministic. Later, this restriction is relaxed to consider "partially" predictable cases.

[174] Technically, the data in Figure 15.2 - 1 a) is only exactly predictable if there is "absolute precision" (e.g. no round-off errors etc), and 1 b) is, strictly speaking, only "pseudo" random.

[175] The "sample" equation used here could have been one of a very large number of equations with the property that the generated "history" is random looking. Some of those equations do a much better job of replicating the market process, but are more complex than is necessary for the present illustrative purposes.