15.3 Spectral Analysis and Periodic Processes

Spectral Analysis is the study of periodicity. The tools from spectral analysis can also assist in characterising whether a process is exactly periodic (e.g. a Sine wave), aperiodic (e.g. chaos and fractals), or “infinitely periodic” (purely random).

It a process or dataset can be shown to be exactly periodic, then it is deterministic and may be (exactly) predictable.

An aperiodic process may be still exactly predictable, or at least permit the extraction of “structure” buried in the data, and which can be used to produce “more accurate” forecasts. In that case, spectral methods may be incomplete and may need the assistance of methods belonging to Non-Linear Dynamics.

Purely random or “infinitely periodic” process may not benefit too much from spectral analysis. In this case, the purely statistical methods, such as those in Chapters 12 - 14 may be the best that one can do.

This Section provides an introduction to:

- Definition and representation of periodic processes.
- Introduction to state space analysis.
- Characterisation of periodic process via spectral and power methods.
- Simple spectral analysis of the S&P.

15.3.1 Periodicity and Aperiodicity

Figure 15.3 – 1 a) and b) show two charts that appear to be “market-like” price histories.

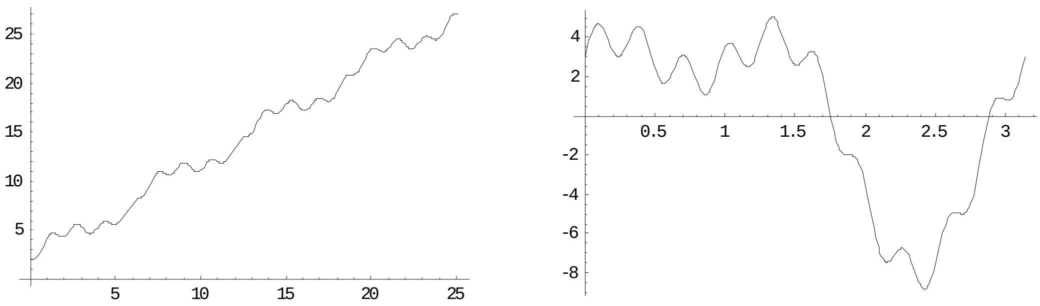

Figure 15.3 – 1 a) & b). Two exactly periodic “market-like” histories.

If the fundamental nature of the market histories were in fact exactly (though possibly complicated) periodic patterns (such as those in Figure 15.3 – 1), then spectral methods (such as Fourier series introduced in Chapter 8.1.2) could be used to exactly predict forward prices.

15.3.2 Representations of Periodicity

This Section provides a review of standard periodic processes and their properties. Applying such methods both to predictable and to "market-like" histories will help illustrate how and why the tools from spectral methods and NLD arise and may assist in improving trading and risk management.

A simple illustration of a "usual" periodic system[182], a sine curve, is provided in Figure 15.3 – 2:

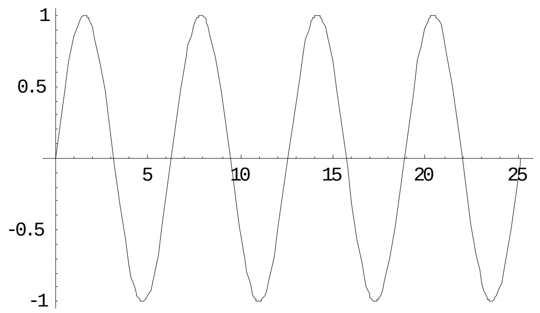

Figure 15.3 – 2 a). A standard Sine curve (P = Sin(t))

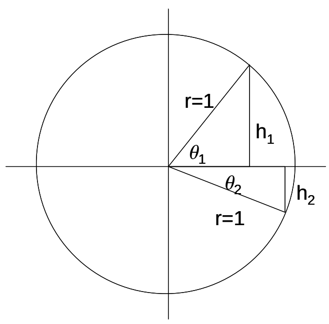

Figure 15.3 – 2 b). The "phase view" of Sin(t).

Now recall that *Sin*(θ) is the ratio of the "opposite" length of a right triangle in relation to the hypotenuse. The Figure 15.3 – 2 b) shows two possibilities for the case that the hypotenuse lies on a unit circle, and is also the radius *r*. It can be seen that the sine curve has a relationship to the circle: every point on the circle can be expressed as $Sin(\theta_i) = h_i/r$. This shows a connection to periodicity in the sense that θ (the "phase angle") repeats itself every time the circle is "lapped". Here θ can be used to describe the "period" or "frequency" of the system since each "lap" around the circle is equivalent to a single "wave" in Figure 15.3 – 2 a).

---

[182] The expression "normal mathematical methods" or "usual periodic processes" are used to refer mathematical problems that are differentiably smooth, or are Lipschitz continuous, or are Riemann intergrable, as per the methods in Chapters 3 and 4. A key feature of "normal" mathematics in the present context is "mathematical smoothness", which refers to the continuity of the differentials (rather than any issue to do with the "appearance of smoothness". Additionally, and as noted previously, there can be dramatic differences in the behaviour of continuous vs. discrete processes as shown with the Logistic equation in Section 15.2.2 and detailed in [3.f].

Now consider that the sine curve in Figure 15.3 – 2 a) and its “phase view” in Figure 15.3 – 2 b) of the sine of an angle are really just two “views” of the same larger picture taken from the “side” and from the “end”, as shown in Figure 15.3 – 3 below:

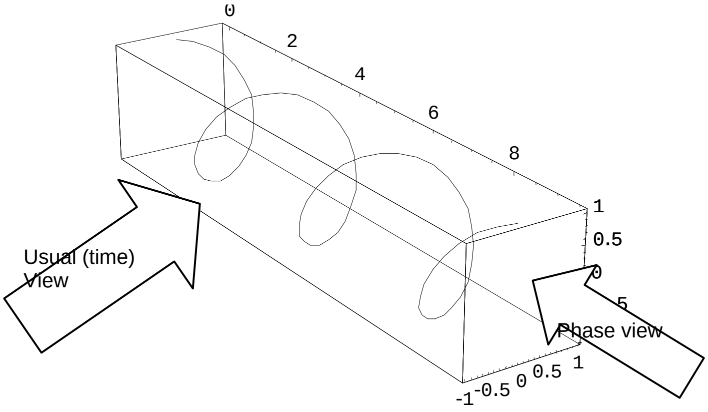

Figure 15.3 – 3. A standard sine curve shown in “full”

Similar considerations can be given to more complex formulations, consider for example a two-cycle periodicity produced with:

$$P(t) = Sin(2t) + Cos(4t) \qquad (15.16)$$

is seen to be:

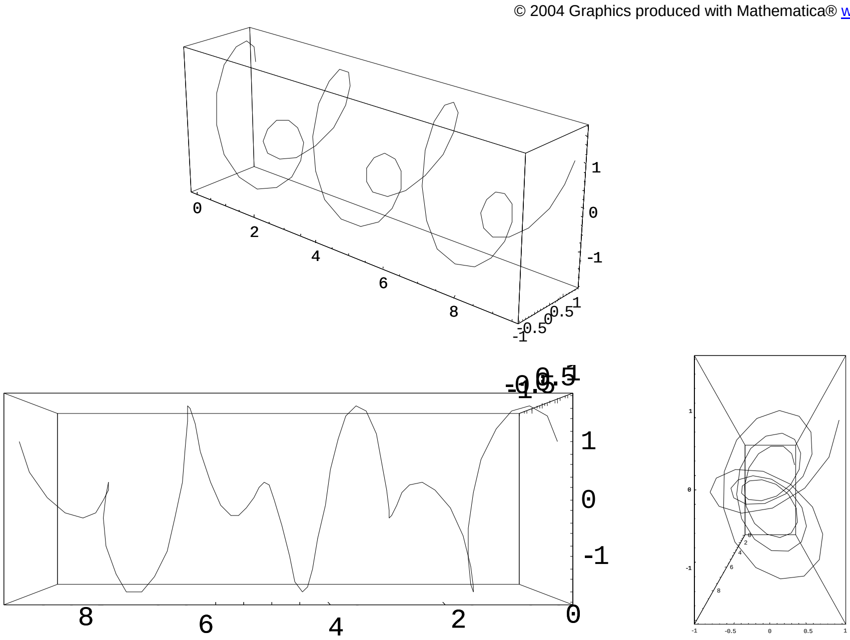

Figure 15.3 – 4 a). Three views of a “two-cycle” process

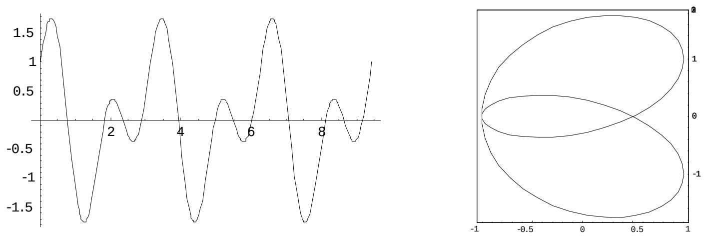

Figure 15.3 – 4 b). Two views of a “two-cycle” process – “perspective removed”.

The phase view in Figure 15.3 – 4 b) clearly shows that there are two different “cycles”, and the phase views will become increasingly useful when assessing increasingly complicated histories.

A periodic process may be in conjunction with other “forces” such as the “damped harmonic oscillator” shown in Figure 15.3 – 4 c).

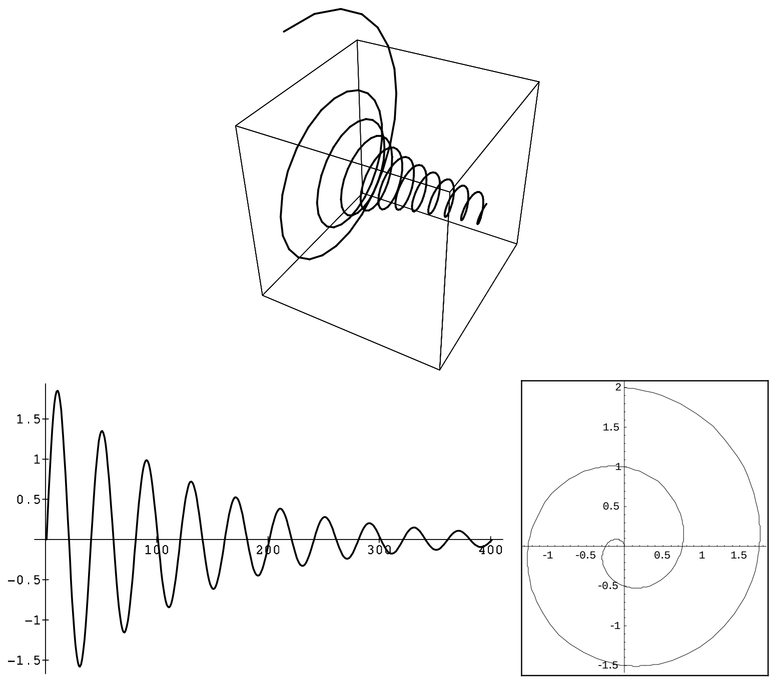

Figure 15.3 – 4 c). View of a damped harmonic oscillator with a single frequency (a damped 1-cycle) ”

The damped oscillator in this example is just a 1-cycle (i.e. a singe frequency periodic function) with an exponential factor, such as: