16 Introduction to Optimisation

Everybody wants the "best". Unfortunately, words such as "optimal", best", and even "sufficient" rank just behind the words "arbitrage" and "volatility" for their misuse and misapplication. Much care is required with these words as they not only require very considerable mathematical/technical machinery, but also have subjective dimensions to them and so mean something different to everyone and every business mandate.

It cannot be emphasised too strongly that "optimal" to a mathematician, almost surely, means something rather different to what a trader has in mind. A mathematically optimal solution is one that has a "minimum[205] distance" between the current and the best answer – within the framework of that mathematical optimiser's assumptions.

In trading, there are subjective aspects of each trader's risk preferences or mandate restrictions, and "physics" that cannot easily (if at all) be incorporated into mathematical optimisation machinery. As such, the larger question of "optimal risk/return over a holding period" is considered intractable by purely (closed form and the like) mathematical optimisation methods.

However, much can be achieved for "partial" problems (e.g. a single rebalance) and even for holding period optimal strategies via simulations that incorporate human judgment to manage the subjective components. In these cases, "optimisation" is often a 2-step procedure with two "optimiser": first, apply a mathematical optimiser to produce a set of outcomes, and second, have the user select the best one based on their preferences.

As with stochastic calculus, the mathematic of optimisation is just too difficult and lengthy for a complete treatment in an introductory text. Here, the primary objectives are to provide a sufficient understanding of the key methods to permit qualitative and business level discussions/decisions, and the ability to obtain solution of simpler problems. Indeed, just selecting the "best optimiser" for the problem at hand already requires non-trivial knowledge and tools.

Additionally, there is a temptation to jump into the deep end and start generating optimal solutions with advanced mathematical methods (since sometimes there is the perception that "complex must be better"). Before you do, though, have a look at the very easy methods, such "rank" based "optimal", just in case the easy answer is also the best answer.

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[205] Optimal can also the "maximum" distance, but in a mathematical setting it is always possible to take the "negative problem" and restate it in terms of "minimum distance".

This Chapter is divided into two parts:

- Mathematical optimisation – basic methods
    - Rank Optimal
    - Linear Programming
    - Quadratic Programming
    - (General Non-Linear) Dynamic Programming

- Trading optimisation – application of basic methods
    - Single rebalance: Hedging
    - Single rebalance: Asset allocation with MVO
    - PaR Simulations: holding period optimal risk-adjusted P&L

Finally, all application of any of these "optimal" trading decisions will almost surely imply one or more modelling assumptions. These assumptions may or may not approximate "your world" sufficiently well, and so P&L based "proof" should be obtained whenever possible or appropriate (such as with the simulation methods in Section 16.4.4 and trade audits).

Not surprisingly, "optimal" trading and risk issues are necessarily the key theme throughout this Series. Portfolio specific matters are introduced in [1], with detailed treatment in [11].