17 A First Description of Four Important Numerical Methods

The two primary modelling problems/equations throughout trading and risk management are the forward price (*P*) forecast equation and the "hedged position[232]" equation for a derivative or security (*f*), often written as:

$$dP = \underbrace{P(r - q - \frac{\sigma^2}{2})dt}_{drift} + \underbrace{P\sigma_r dz}_{uncertainty} \qquad (17.1)$$

and

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial p^2}(\sigma p)^2 + \frac{\partial f}{\partial p}(r-q)p = rf \qquad (17.2)$$

This Chapter introduce several numerical methods for solving these types of Partial Differential Equations (PDEs) numerically. The solution of DEs is closely related to integration and summation, and the numerical methods reflect this.

It cannot be emphasised too strongly that the methods discussed here "solve" model equations, and do not represent, in themselves[233], modelling as such. These "mechanical process" are generally approximations to the closed form solutions, which would be desirable but may be too difficult or expensive to obtain.

This is a very complex area and requires considerable expertise in general. In many ways, this is an area where "a little knowledge can be a dangerous thing". Thus, the primary objective is to provide sufficient information so that you can solve simple problems on your own, but mainly in the context of coming to understand the methods and their properties. This should permit "business level" discussions/decisions when implementing such tools.

Roughly speaking numerical strategies fall into two classes: "simulation-based" and "PDE-based" methods. The three primary methods introduced here, with sample implementations and calculations, are:

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[232] Here, the equation is written in terms of its "risk neutral" form. However, the solution methods of this equation also provide solutions for the "risky" version (i.e. one that includes a risk premium such as $\mu = r_{RF} + \lambda\,\sigma$)

[233] Importantly, some methods, such as the "binomial model" can be considered both "models" and "solutions methodologies", but not both. Here, the binomial method is used ONLY in the context of an approximation to the PDE, and not as a model in itself.

- Monte Carlo Methods (simulation-based)
- Tree/Lattice Methods (simulation-based)
- Finite Difference (FD) Methods (PDE-based)

One or more of these three methods should be your first choice for solving almost any PDE related problem in trading and risk management.

Two other methods are included, primarily to provide an understanding of "what they are", and their strengths/weaknesses, these are:

- Finite Element (FE) Methods (PDE-based)
- Boundary Elements (PDE-based)

These are very much more technical in their nature, and they offer special advantages when applied to certain types of problems. These special circumstances arise in finance from time to time, but are more common in other industries (such as civil engineering, and oil exploration). Outside of those situations, they are generally less effective for trading and risk management than the 1st three methods listed above.

Each of the methods introduced here is capable of solving a large range of pricing and hedging problems, and the methods "overlap" considerably. For the most part, the decision relating to the method of "choice" will be subjective, but the Tree and MC methods tend to be easier to understand, and much less expensive to implement. On the other hand, while the PDE-direct methods (such as FE and FD) are more expensive to implement, they are better suited for complex problems, and offer much greater efficiency and can be used to create a comprehensive valuation/risk management framework.

Finally, numerical methods for PDEs are bit a like a "do it yourself brain transplant kit". It sounds good "on paper", and it appears as it may be something you could do yourself, but much can go wrong, and its best left to the experts.