3.5.2 Total Derivatives

It is possible to describe the changes in the elevation on the roof by breaking down the "components" of any general motion into the components along the coordinate axis that arrive at the same point.

$$D[elevation] = \frac{\partial V}{\partial x} d[North/South] + \frac{\partial V}{\partial y} \; d[East/West] \qquad (3.21)$$

The notation has changed from using purely *d's* to denote differentiation/tangent, to using a ∂ to denote differentiation in a "multi dimensional" setting. Since there are component slopes in multiple coordinate directions, any one slope is just a "partial" slope. Thus, the ∂'s denote "partial derivatives" or "partial differentials". The *d's* here actually represent the size of the increment[33] in the respective directions.

This expression can be interpreted as follows: The total change in elevation is the (partial) change in elevation due to movement North (or South if –ve) by an amount $d[North/South]$ along a direction that has a slope of $\frac{\partial V}{\partial x}$, plus the (partial) change in elevation due to a movement East (or West if –ve) by an amount $d[East/West]$ along a direction that has a slope of $\frac{\partial V}{\partial y}$, as per Figure 3.5 – 2.

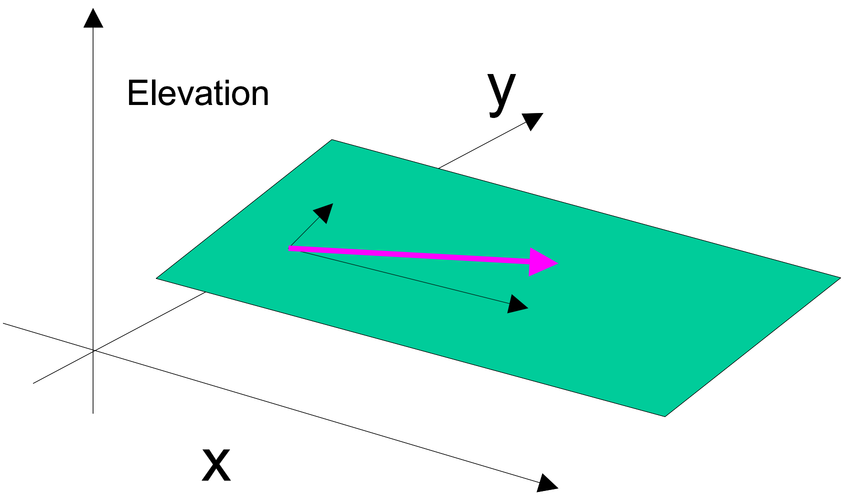

Figure 3.5 – 2. A slope on a surface resolved into component vectors in the coordinate directions.

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[33] Since these are tangent planes, the "size of the increment" is "infinitesimal". In some setting settings the *d's* will be replaced with Δ's to imply "finite sized increments" (such as in V01 etc).

Figure 3.5 – 2 has used a "flat" roof for illustration, but the expression holds true for any "curvature" so long as it is consistent with the rules of the calculus. While Figure 3.5 – 3 aims to illustrate the component coordinate directions of the tangent plane to a curved surface.

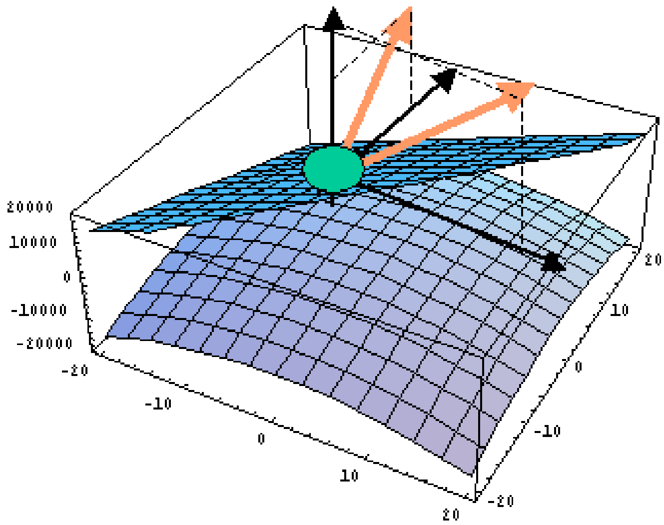

Figure 3.5 – 3. A tangent plane on a surface resolved into component vectors in the coordinate directions in 3-space.

Equations such as Equation (3.21) are called "total derivatives" since they measure the total change (not slope) in a process (at least locally, or at a point).

Total derivatives need not be restricted to "single family derivatives". One of the most important expressions in finance is can be written in total derivative form as:

$$DV = \frac{\partial V}{\partial z} dz + \frac{\partial V}{\partial t} \; dt \qquad (3.22)$$

Equation (3.22) represents the change in position value V for a contract that depends on a change in time t, and a change in "uncertainty" z. The more common form of this expression is via the partial differential equation for say a stock price P that has a "drift" or "growth" (partial) slope of rP, and a partial "uncertainty slope" of σP when there is (an infinitesimal) change or increment in time of dt and (an infinitesimal) change or increment in uncertainty of dz.

$$dP = rPdt + \sigma Pdz \qquad (3.23)$$

Finally, total derivatives are not restricted to 2-D, they generalise to n-dimensional problems with partial differentials and increments for each required coordinate direction.