4.2.2 The "area under the curve again" – CumNorm(x) & Cos(x)

This Section provides two examples of integration with graphical illustration of the relationship between the "integrand" and the area under the curve (i.e. the integral).

4.2.2.1 The Normal Distribution and the Cumulative Normal Distribution

One of the most important integrations in finance and trading is that of assessing the "total" or "interval" probability or likelihood of the occurrence of an event such as the chance of having to make a pay-out at a contracted strike price, or the likely drawdown of loss in the P&L of a portfolio during uncertain markets. The likelihoods are commonly "modelled" by way of distributions (see Chapters 8 and 11), and by far the most common probability distribution in use is that of the Normal distribution (the classic "bell curve" as in Figure 4.2 – 2).

However, the actual probability required for an options strike, or for the VaR of a portfolio is related to the "interval" probability between, in this case, two price or P&L levels. For example, in the case of a call option the likelihood of a pay-out against the strike is related to the chance of any price at or "above" the strike. Such interval probabilities are obtained by summing all of the "individual " probabilities in the over the interval, which for continuous functions, is arrived it by integration (i.e. the area under the curve).

Suppose that the probability function is given by:

$$P(x) = \frac{1}{\sqrt{2\pi}} e^{(x-\sigma)^2/2} \tag{4.37}$$

then the interval probability, say between $-\infty$ and x is:

$$CumP(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{\left(\frac{(x-\sigma)^2}{2}\right)} dx \tag{4.38}$$

Figure 4.2 – 2 shows plots of the probability function as well as the area under the curve, for several choice of x

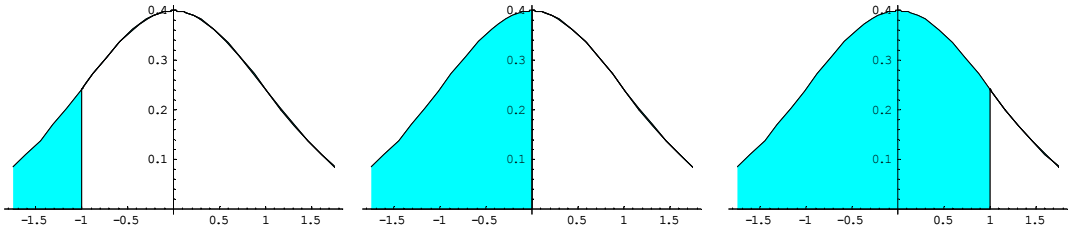

Figure 4.2 – 2. The Normal distribution with increasing intervals relating to increasing area under the curve

It is also possible to plot just the values of the areas, which for this set of choices for x is shown in Figure 4.2 – 3 as the three points corresponding to the three areas.

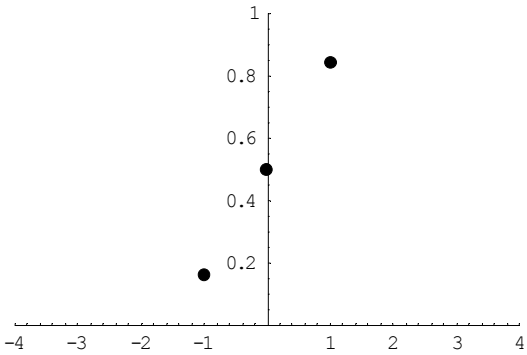

Figure 4.2 – 3. Three points representing three areas under the curve.

Figure 4.2 – 4 a) and b) shows the same process for very many such evaluations, leading to a full or continuous "curve" of "Cumulative Probabilities" in b).

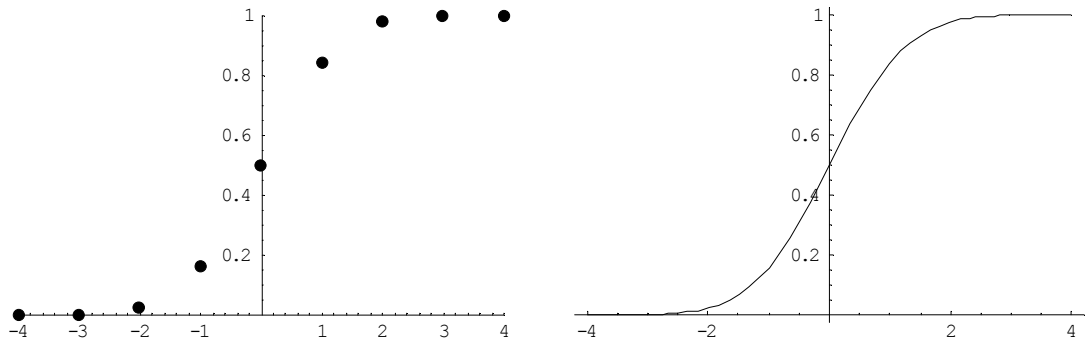

Figure 4.2 – 4. a) Many points representing many areas under the curve, and b) a continuum (function) of areas under the curve: the cumulative probability function