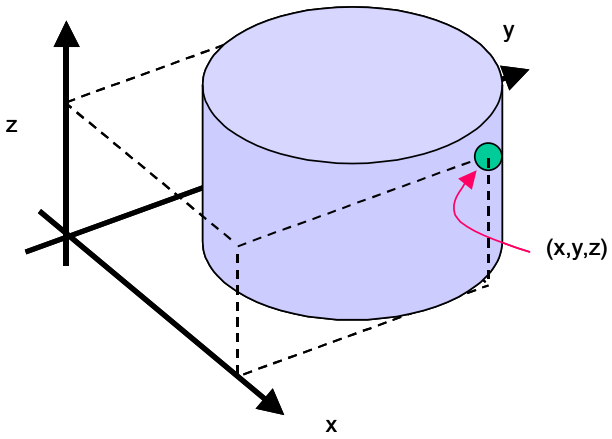
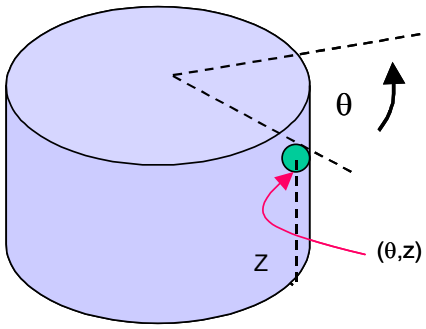
7.5.3 Eigenvalues and Eigenvectors

Eigenvalues and eigenvectors relate to the "proper" or "natural" coordinate system of the problem represented by the system of equations. Important, they relate not only to the minimum number of coordinates, but also provide expressions for the "key" parameters that describe the process at hand. Similarly, they can be used to assess important features of flows and dynamical systems. For instance, they relate to the natural coordinates in a transition matrix, such as might be used for forecasting yield curves, as in Chapter 17.

For example, it is common to express coordinates in 3-D Cartesian terms, since that is what we experience for the most part in the real world. Now, suppose that you had to provide an address for somebody living on the surface of cylindrical block of flats[332]. The image to the right shows a cylindrical surface embedded in a 3-D Cartesian coordinate system. The "address" of the person living at the flat denoted by the green dot is the point *(x,y,z)*.



However, notice that the geometry of this block of flats is cylindrical, and so the address of the same person can be expressed in "cylindrical coordinates" as *(θ,z)*, where θ is the angle of rotation from some fixed point (i.e. the equivalent of zero).



Notice that the cylindrical coordinate system requires only two coordinates to uniquely specify the "address", while the Cartesian system required three coordinates.

The minimum number of coordinates should be two, since this is a problem on a surface, and a surface is a 2-dimensional object.

Therefore, the number of coordinates used may not be the same as the minimum number required. Moreover, that this difference may be purely a matter of convention.

Consider modelling the dynamics of a yield curve, or the modelling of profitability of asset allocation/basket trading. In both cases, it is possible to choose haphazardly the number of and specific "constituents" of the curve or basket. Now, one may wish to ask "what is the

[332] A "block of flats" is also called an "apartment building" by some.