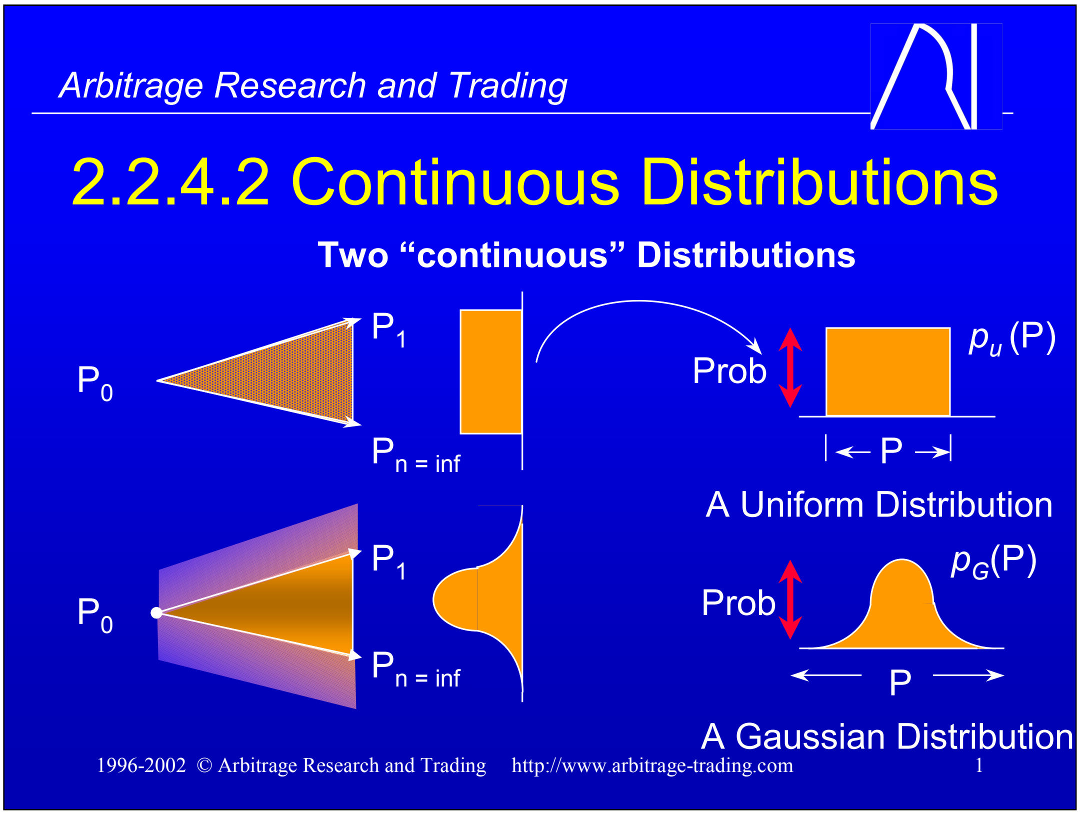
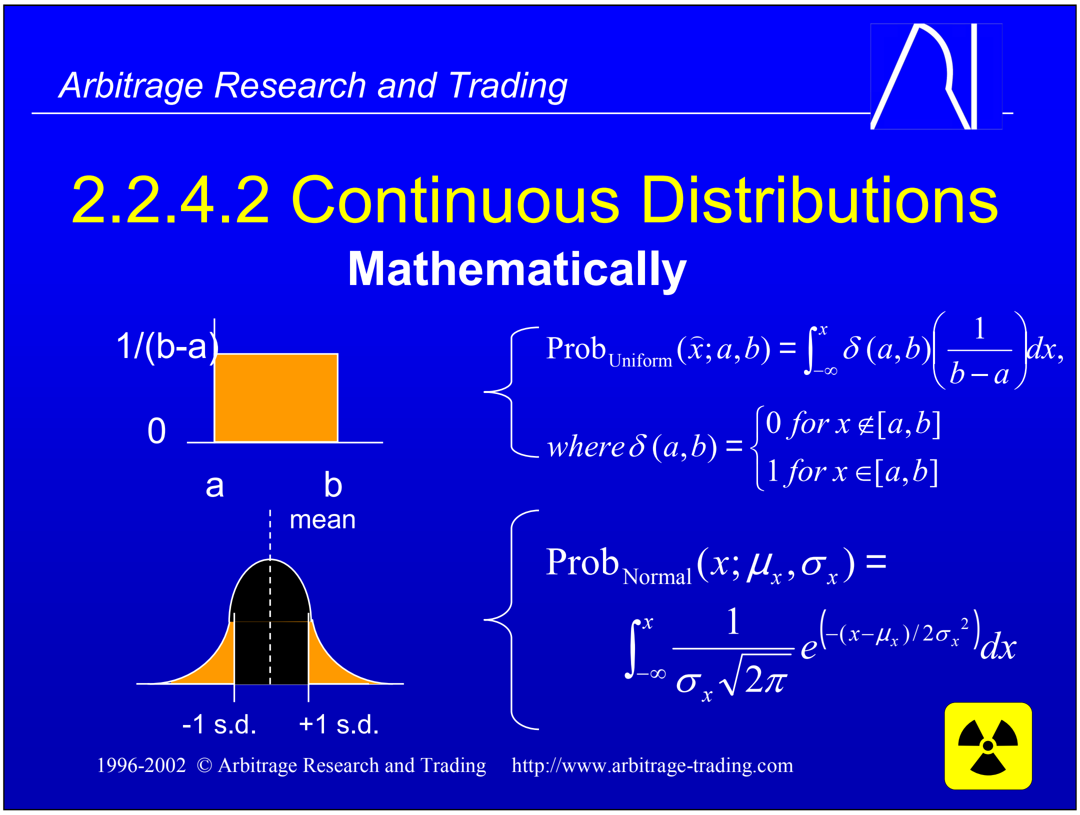


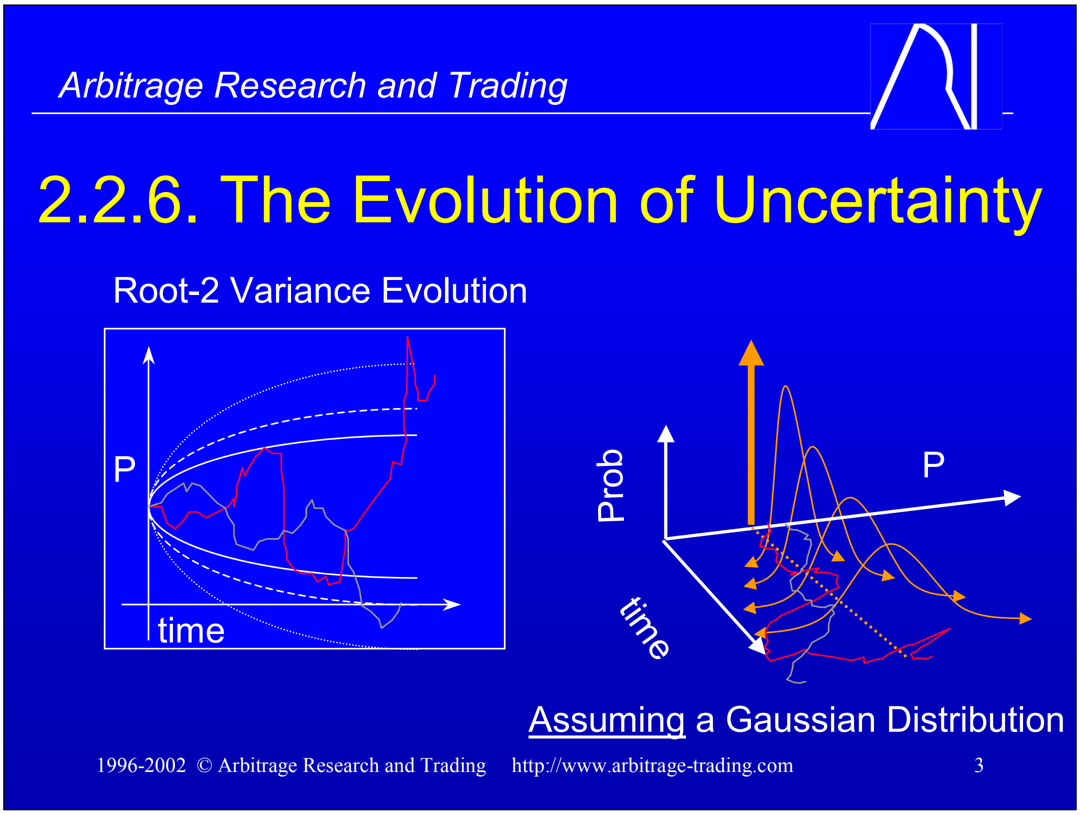
At the early stages of illustrating "OptionsMaths" there a number of slides that show, pictorially, assumptions we can make about the shape of the forward price or rate distribution on any given forward date. This particular slide shows two of the possible distributions.

Related slides illustrate that the starting point is usually the Gaussian distribution, though in some cases the Gaussian distribution is quite a bad model of forward, and later sections delve deeper into those problems

Importantly, it is also emphasised that in one sense the "correct" distribution is the one that most closely suits your particular business mandate. For example, a prop trader requires a distribution with the best "trading P&L", while market makers require distributions that also permit the "flows" (e.g. must have some connection to market convention). In other sections, methods are introduced that further help assess how to chose the "best P&L" based approach. We also provide approaches that help to reconcile differences between trader's preferences vs. back office requirements and reporting/regulatory issues, in a manner that still permits everybody to do their job.



With “OptionsMaths” there is invariably some “maths”. The presentations include these for completeness and for those who require the equations. However, in many cases it is emphasised that traders may wish to not spend too much time learning how to “derive” equations, but rather to understand the qualitative implications of the model (e.g. via the “pictures” and worked examples). To highlight this issue, slides that are not “essential” for traders have the “yellow radiation hazard” symbol.



The early portion of “OptionsMaths” also illustrates that the forwards distribution are required for each future date where there is an “event” (e.g. cash flow, rebalance, etc). This essentially results in a “mountain range” of distributions, where the distributions going forward in time are “stretched” to reflect the increasing uncertainty about the forward prices/rates.

This slide illustrates the mountain range for the Gaussian/Root2 process embedded in virtually all options and related type calculators in common use.

In other sections, it is shown that different mountain ranges are more P&L consistent due to either or both of market conditions or instrument idiosyncrasies.

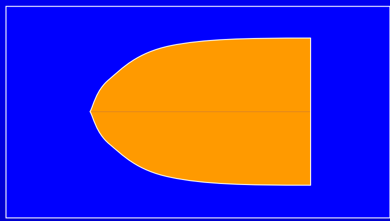
Arbitrage Research and Trading

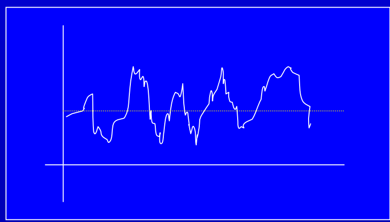
4.3.3. A mean reverting processes

$$dP=\kappa(\psi-P)dt+\sigma P^{\omega}dz$$

This is called the Ornstein-Uhlenbeck equation

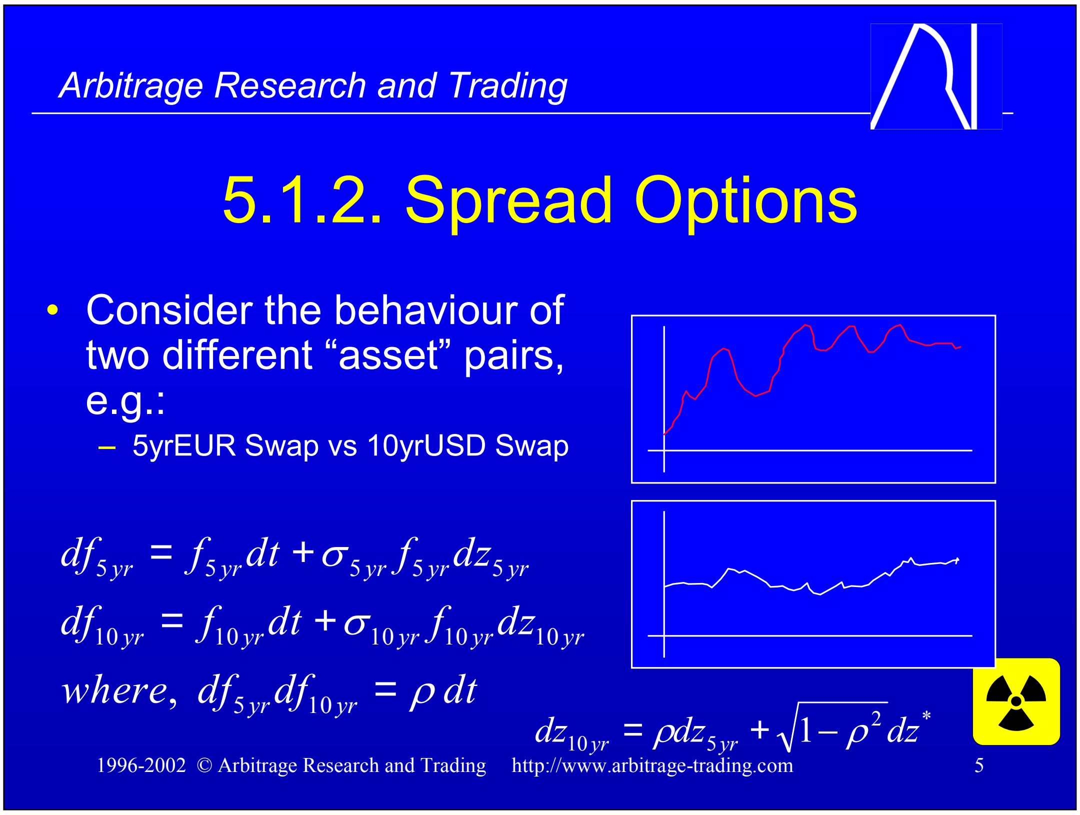
- Mean reversion implies P fluctuates about its long run mean (ψ) with reversion speed (κ)
- proportional volatility with extra fine tuning parameter (ω)



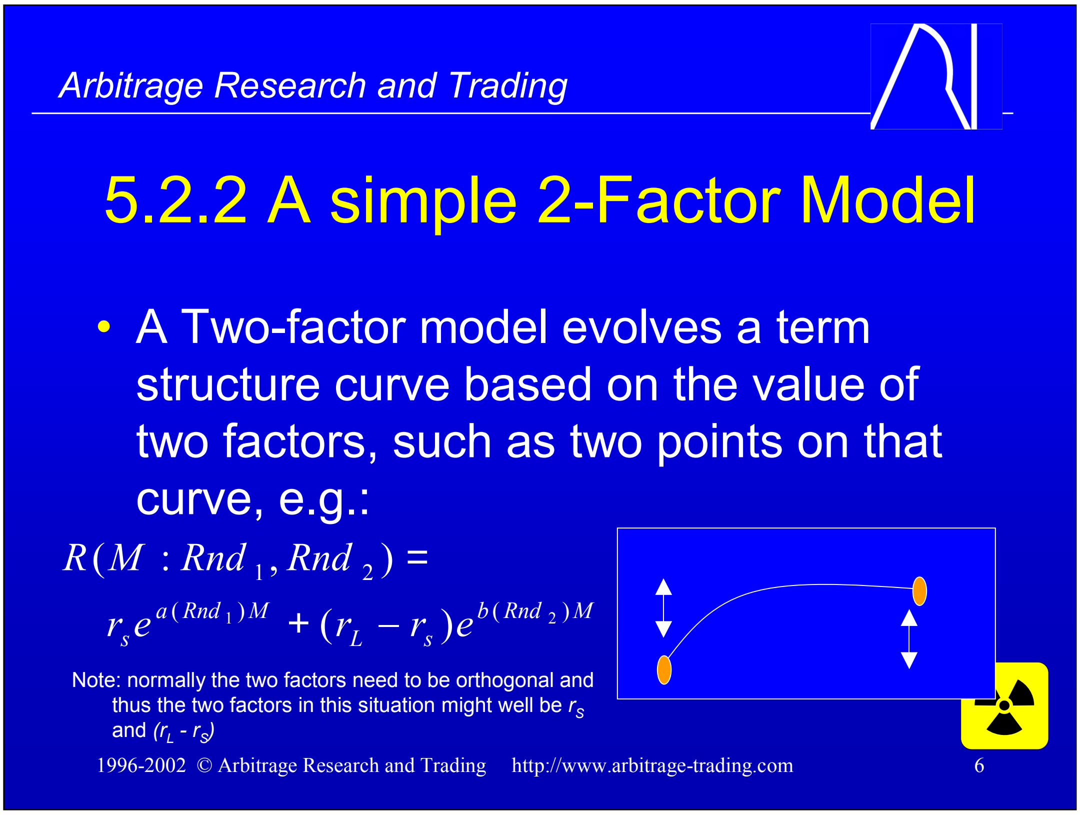


 4

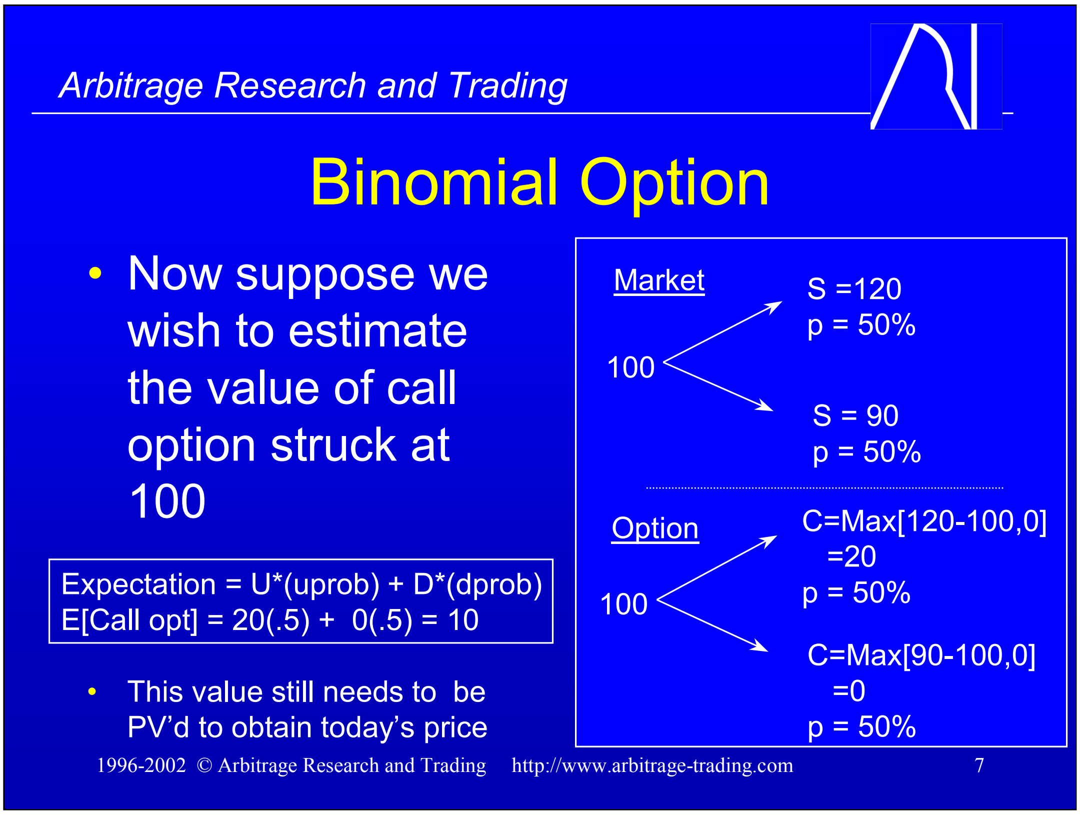
In all cases, the "OptionsMaths" are always presented at several level and in each case "qualitative" and "pictorial" metaphors are included to show the implications.



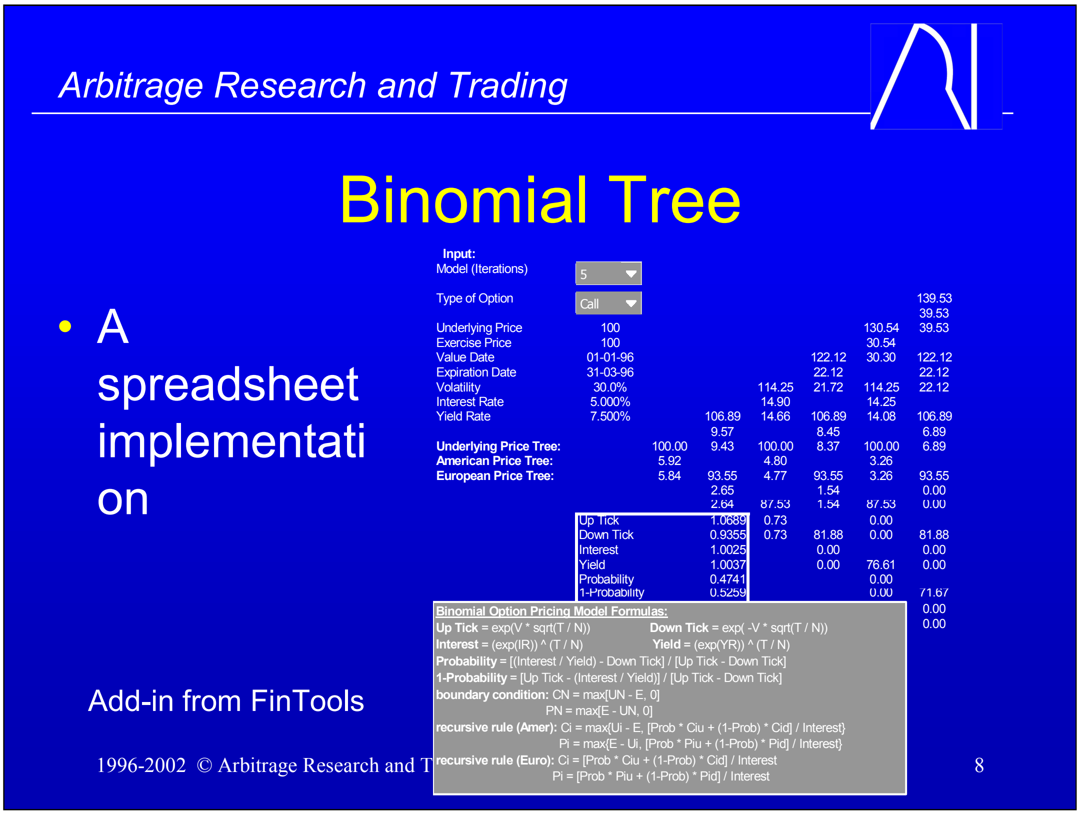
There are many slides available for the valuation of complex structures, such as this slide taken from a series on the valuation of “cross currency calendar swap spread options”. Other slides show easy methods for implementing this problem in XL spreadsheets and other calculators.



Many well know sophistications are presented, such as 1-factor, 2-factor, and n-factor models for term-structure problems. However, and as this slide illustrates, we also generate additional "concocted" models which are not exactly the market convention, but they do make it easier to understand how and why such models work, and what it may mean to your P&L and risk.



Many techniques are illustrated from the most basic level to the more sophisticated implementation. This slide is one of many on the use of binomial trees for implementing the options models.



The illustrations contain many slides with increasing level of sophistication. Additionally, we also show you how you could do this yourself either by implementing it directly, or by simply purchasing/using off-the-shelf tools.

Arbitrage Research and Trading

Finite Difference methods

- following appropriate substitutions we obtain a (local) algebraic equation

$$\frac{1}{2}\sigma^2 S^2 \frac{(V_{i+1} - 2V_i + V_{i-1})}{(\Delta S)^2}\bigg|^{n-1} + rS\frac{(V_i - V_{i-1})}{\Delta S}\bigg|^{n-1} - rV_i\big|^{n-1} \cong -\frac{(V^n - V^{n-1})}{\Delta t}\bigg|_i$$

$$-\Delta t[aV_{i+1} + (-2a + b - r - \frac{1}{\Delta t})V_i + (a - b)V_{i-1}]\bigg|^{n-1} = V^n\big|_i$$

$$a = \frac{1}{2}\frac{\sigma^2 S^2}{(\Delta S)^2}\bigg|_i^{n-1} \qquad b = \frac{rS}{\Delta S}\bigg|_i^{n-1}$$

 9

Advanced methods for implementing options valuation are also covered, and include not only explanation and examples but also free downloadable software. For example, this slide is one of the many slides on using Finite Differences to value options. This and related slides show that all the FD method does is to convert the options model's (partial differential) equations into a set of algebraic equations based on deltas, gammas, and thetas.

FD methods

- a spread sheet implementation of the computational stars and ICs,
- the BC's were calculated as extrapolations of the slopes at the upper and lower edges

vol	35.00%
ir	10.00%
dS	10
dt	-0.0417
Strike	100
DPY	365

Days to Expiry

Underlying	15.21	30.42	45.63	60.83	76.04	91.25	106.46	121.67	136.88	152.08	167.29	182.50	
	51.47	52.01	51.27	51.65	51.04	51.29	50.77	50.90	50.41	50.42	50.00	50	
140	43.44	42.85	42.86	42.33	42.26	41.77	41.64	41.19	41.03	40.62	40.42	40	140
130	34.51	34.23	33.72	33.38	32.87	32.50	31.99	31.61	31.15	30.83	30.42	30	130
120	26.04	25.49	24.99	24.41	23.85	23.24	22.65	22.02	21.42	20.83	20.42	20	120
110	18.03	17.44	16.81	16.16	15.47	14.76	13.99	13.19	12.32	11.39	10.42	10	110
100	11.10	10.49	9.86	9.20	8.50	7.76	6.96	6.09	5.13	4.00	2.55	0	100
90	5.74	5.23	4.70	4.16	3.59	3.01	2.41	1.79	1.15	0.53	0.00	0	90
80	2.30	1.97	1.64	1.32	1.02	0.73	0.47	0.25	0.09	0.00	0.00	0	80
70	0.63	0.49	0.36	0.25	0.16	0.09	0.04	0.01	0.00	0.00	0.00	0	70
60	0.10	0.07	0.04	0.02	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0	60
50	0.01	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	50
40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	40
30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	30
20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	20
10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	10
0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	0
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0	

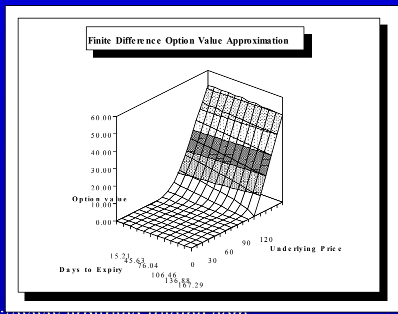
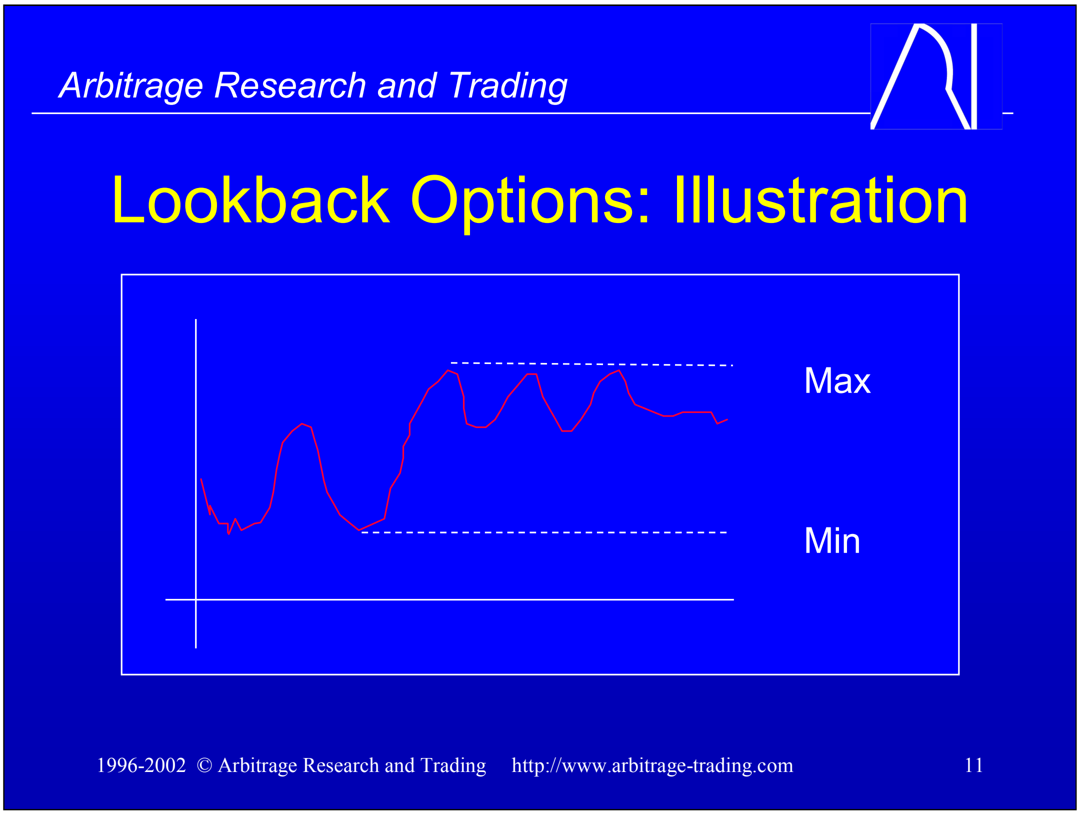


Figure 6. Explicit Finite Difference approximation to the Black-Scholes PDE

The supplied spreadsheets implement the FD method directly in the "cells" and so there is no programming to unravel, just simple look at the cell contents, or just use it to start testing the FD method as a risk/valuation tool.



Other section of the "OptionsMaths" cover virtually any complex product that is of interest. In each case there are qualitative explanations of the payout structure, as well as economic need for supply/demand and end user requirements considerations.

Lookback Options: Uses

- Hedging:
  - e.g. have to make payment in arrears based on early FX translation
- Investment:
  - punting tool?? probably too expensive
- Expect it to be expensive

Here is one slide that highlights some of the “tips&tricks” for Lookbacks.

Lookback Options: Call Pricing Formula

$$Call_{Loockback} = Se^{-qt}N(a_1) - Se^{-qt}\frac{\sigma^2}{2(r-q)}N(-a_1)$$
$$- S_{\min}e^{-rt}\left[N(a_2) - \frac{\sigma^2}{2(r-q)}e^{Y_1}N(-a_3)\right]$$

$$where, \quad a_1 = \frac{\ln(S/S_{\min}) + (r-q+\sigma^2/2)t}{\sigma\sqrt{t}} \qquad a_2 = a_1 - \sigma\sqrt{t}$$
$$a_3 = \frac{\ln(S/S_{\min}) + (-r+q+\sigma^2/2)t}{\sigma\sqrt{t}}$$
$$Y_1 = -\frac{2(r-q-\sigma^2/2)\ln(S/S_{\min})}{\sigma^2}$$

Note: formulation from Goldman,Sosin, and Gatto and from Garman, using Hull's notation

Other slides are included which provided references to published works. Though of course we also provide other methods for implementing the models such Monte Carlo, Tree, and so forth.